OPTIMIZATION OF SPATIAL MECHANISMS

By

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With love to Jeri and Nicholas

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The material in this dissertation can be effectively divided into two subtopics: philosophy of optimal mechanism design, and optimization of dyad-based spatial mechanisms.

The first subtopic, philosophy of optimal mechanism design, is intended to be general in nature, applying to all types of mechanisms, both higher and lower pair, and both planar and spatial. This is covered in Chapters One through Three. Chapter One examines past approaches to mechanism optimization. Chapter Two is a brief review of optimization theory, particularly as it applies to mechanism optimization. Chapter Three draws upon the insights gained in the first two chapters to formulate a general approach to the mechanism optimization problem.

The second subtopic of this dissertation, optimization of dyad-based spatial mechanisms, is covered in Chapters

Four through Seven. This is actually a rather limited example of applying the philosophy developed in the first three chapters. Nevertheless, the mechanisms treated in this section are believed to represent some of the most useful motion generating spatial mechanisms, and, therefore, those for which improved design theories are most urgently needed. In Chapter Four, closed-form synthesis equations are derived for dyads containing revolute (R), spheric (S) and cylindric (C) pairs. Chapters Five and Six present detailed examples of the optimization of the four-link RCCC and five-link RSSR-SC and RSSR-SS mechanisms. Finally, Chapter Seven outlines procedures for the optimization of other dyad-based spatial mechanisms, and offers suggestions for further research.

CHAPTER 1

BACKGROUND AND MOTIVATION FOR THIS RESEARCH

1.1 Introduction

The design of mechanisms, like almost all design problems, is an iterative process. Generally, a mechanism can be synthesized to meet some of the design requirements, but then it must be analyzed or tested to determine whether or not it satisfies the remaining requirements. Most often it does not, and the designer must "go back to the drawing board" and synthesize a new mechanism which again must be analyzed or tested. This second design may also not satisfy all the design requirements, but it is likely to be somewhat of an improvement over the first design, since the designer had the "experience" of the first design on which to base this second synthesis. The third design attempt will likely be better still, since now two points of reference are available. This iterative process is continued until a satisfactory solution is obtained. It is probably the oldest and most often used form of optimization.

For the purpose of this dissertation, optimization may be considered to be the process of seeking the best result under a given set of circumstances. The questions of what optimization processes are available, what is meant by the "best" result and how these "circumstances" are incorporated into the solution will be topics of considerable importance. The following paragraphs briefly introduce these concepts.

As has already been stated, optimization may be any process which seeks to find the best result to a given problem. Available optimization methods include the intuitive successive improvement approach described previously; classical optimization methods, such as the Lagrange multiplier method, based on the principles of variational calculus; and a large and rapidly expanding body of knowledge known as mathematical programming. The last of these, mathematical programming, is most important with respect to this work. In a sense, mathematical programming methods are an extension and formalization of the age-old successive improvement process. These newer methods, though, have become quite sophisticated, employing a variety of numerical techniques and almost always requiring programming on a digital computer. Also, by formulating some measurement of the quality of a given design, the burden of decision-making after each iteration can be removed from the

designer. The most general form of mathematical programming aims at solving nonlinear types of problems and, not surprisingly, is called nonlinear programming. It will soon become evident that mechanism optimization generally involves nonlinear programming. A detailed discussion of optimization methods is given in Chapter Two.

The second question to be addressed is what is meant by the "best" result. Suppose, for example, that an airplane strut is to be designed for a given design load with maximum stiffness, but with minimum weight. It is apparent that these represent competing design requirements, and that some compromise between the two must be reached. Here, the designer's judgment must enter the process and he must decide what importance or "weight" to assign to each requirement. Of course, mechanism design problems are rarely this simple, and will usually involve a much larger number of design requirements. As a result, finding the best design usually involves some amount of subjective opinion. Even if the importance of the various design requirements were exactly known, the best mathematical solution would probably not correspond to the best physical solution. This is because there are inherent errors and approximations present in the modeling and analysis of any real system. Thus, although mathematicians may speak of a "globally optimum"

or best solution, the best that can be hoped for in an engineering setting is to find some approximation to the best solution.

The final topic of this section addresses the question of what role the "circumstances" play in determining a best design. Will the machine designed to run at 60 revolutions per minute (rpm) perform acceptably at 600 rpm? Perhaps not, since the circumstances or "constraints" under which the design was developed have been altered. Constraints may be cast in the form of equality or inequality conditions, and they may be explicit or implicit. In any case, it is evident that the circumstances surrounding a design will also have an effect on which solution is best.

1.2 The Elements of Mechanism Optimization

Generally speaking, the optimization of any system or component comprises four principal elements. These are

(1) the choice of a conceptual design;

(2) the development of an analyzable model of the physical system from which the design variables and the governing equations may be extracted, both for synthesis and analysis;

(3) the setting up of a scalar function (called the objective function) of the design variables which serves as a

measure of the effectiveness of the system, accompanied by specifying all design constraints; and finally,

(4) finding the set of values of the design variables which produce the best value of the objective function of part (3) and are consistent with all design constraints.

The first three of these elements are primarily dependent on the specific problem at hand; they will collectively be called the problem formulation. The fourth element is purely an application of optimization theory. One of the primary objectives of this dissertation is to develop a method of mechanism problem formulation in such a way as to accurately reflect the problem requirements, while maintaining a simple enough structure to allow efficient and sufficiently accurate solutions by one of the available optimization methods.

The first element of the problem formulation to be addressed is choosing a conceptual design. The designer must decide what types of mechanisms, such as linkages, cams, belts, gears, cam-modulated linkages, etc., are best suited to solve a particular problem. Following this, he must decide the particular configuration of the

mechanism chosen. If a linkage has been selected, should it have four links or six links? Should it be planar or spatial? The above decisions are known as type and number synthesis, respectively. Although these are extremely important decisions in the optimization process, a detailed discussion of them is beyond the scope of this work. Throughout the remainder of this dissertation, it will be assumed that a specific mechanism type has been selected, and optimization must proceed from this point.

The second element of problem formulation consists of synthesis, modeling and analysis of the system conceptualized above. In the present work, this stage will draw heavily upon the classical theories of mechanism synthesis and analysis. Here the term analysis includes, among other things, application of Grashof's condition (or a similar spatial condition), branch and order determination and, of course, position, velocity, and acceleration analysis. Typically, the mechanism to be optimized will be modeled as having rigid links and zero clearance in the joints, although other assumptions such as elastic links and joint clearances and/or compliances may be applied where appropriate.

The third and final element of the problem formulation is the setting up of the objective function and the design constraints. The objective function is a scalar function of the design variables whose numerical value reflects

the quality of the system being designed. In designing a mechanism for a specified performance, it may be desired to minimize the sum of the squares of the structural errors[1] at a large number of positions. Or perhaps it is desired to minimize the maximum deviation of the transmission angle from the ideal value of 90° throughout the motion cycle. Maybe some weighted combination of these two requirements in the objective function is best. It is evident that the designer must possess a thorough understanding of the requirements of the system being designed if he is to choose an effective objective function.

1.3 History and Literature Review

This dissertation brings together two fields, namely, optimization theory and mechanism science, which historically have a related and remarkably similar development. Both disciplines have their roots in early civilization, although neither was formally recognized until quite recently. Both disciplines were topics of interest and were significantly advanced by the great mathematicians of the seventeenth and eighteenth centuries. Newton, Euler and Lagrange are but a few of the men whose names

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[1]Inherent deviation from the prescribed performance, expressed as a scalar or vector quantity.

are common to both subjects. Even with the contributions of these and many other great men, development of both subjects remained slow and sporadic well into the twentieth century.

By about 1945, both mechanism science and optimization theory had reached somewhat of a plateau. Graphical methods of planar mechanism design and analysis were well established, although their usefulness was limited by the sometimes inaccurate and tedious nature of graphical constructions. Classical optimization methods, based on calculus, were also well established, but were useful for solving only a limited class of problems. Both fields, mechanism science and optimization theory, were advancing at a relatively slow pace. The possibility of solving new types of problems or more difficult problems seemed distant indeed.

Then came the dawning of the computer age. Suddenly, the potential arose for handling large amounts of information, and for performing tedious calculations quickly and accurately. The growth rate of both fields virtually exploded. Kinematicians adopted algebraic methods which were soon translated into computer code. It became possible to generate hundreds or even thousands of mechanisms as possible candidates to solve a given problem. It soon became apparent that the computer could generate more solutions than the designer could reasonably assess.

The next logical step was to let the computer examine the relative merits of each solution, presenting to the designer only those mechanisms which met some preestablished criteria. It was quickly recognized that the rapidly advancing field of optimization theory was exactly the tool needed for this purpose. The premise was a simple one: to allow the computer to perform a logical search for the mechanism or mechanisms which best satisfied the designer's requirements under the given circumstances. This is the foundation of practically all modern mechanism optimization.

The previous section gave a very brief summary of the early history of kinematics and optimization. This section outlines the developments in kinematics since about 1950, particularly as they relate to synthesis and optimization of spatial mechanisms. Many of the references cited here deal exclusively with planar mechanisms. This is to be expected since often, although not always, the development of planar methods of synthesis and optimization have been, or can be, extended directly to spatial mechanisms. Also, the inclusion of these planar references will help to explain how the trends toward many of the currently used methods were established.

Optimization theory, as a subject unto itself, is not directly treated in this literature review. Rather, in the later section on optimization methods, a number of standard references are cited.

The pioneer of modern kinematics in the United States is generally considered to be Freudenstein. His 1955 paper, "Approximate Synthesis of Four-Bar Linkages," (1) probably marks the beginning of the shift in emphasis from graphical to analytical methods. The expression, "Approximate Synthesis," was widely used during this era to denote precision position synthesis to approximate a given function. It should not be confused with its more recent use to denote synthesis of a mechanism which approximately satisfies a large number of precision positions. Freudenstein's later work with Sandor (2) in 1959 employed complex number theory and a programmed IBM 650 digital computer in the synthesis of path-generating mechanisms. This work, perhaps more than any other, marks the marriage of the kinematician to the digital computer.

Other important contributions from this time period dealing with planar mechanisms include the four-bar linkage atlas of Hrones and Nelson (3), the additional works of Freudenstein (4,5) and the combined work of Roth and Freudenstein (6), to name a few. In the latter work, path generating geared five-bar linkages are synthesized using numerical methods and a digital computer. The work

of A.S. Hall (7), along with his organizational involvement in the early "Conferences on Mechanisms" from 1953 to 1962, gave inspiration and insight to many of the researchers who followed him. McLarnan (8) extended the earlier work of Freudenstein to include the synthesis of planar six-link function-generating mechanisms.

Analytical studies of spatial mechanism synthesis in the United States also received attention during the 1950's. Denavit and Hartenberg (9) provided what has become standard symbolic notation for the description of the kinematic properties of lower-pair mechanisms. In a later paper (10), Denavit and Hartenberg extended the precision position approach of Freudenstein (1) to the synthesis of spatial RSSR and RCCC mechanisms, and showed the synthesis equations to be linear up to a limited number of precision positions.

It should be noted that a number of German and Russian scholars were also contributing to the advancement of knowledge on spatial mechanisms at this time. Notable among these are Beyer (11), Dimentberg (12), Novodvorskii (13), Stepanov (14), and Levitskii and Shakvazian (15). For a more detailed review of these works, the reader is referred to the survey articles of Beyer (16), Harrisberger (17) and Yang (18).

Until about 1965, the only spatial mechanism synthesis problems covered in the literature involved

coordinating motions of input and output links, or as it is commonly known, the function generation problem. Wilson (19) changed this trend by introducing the problem of spatial rigid-body guidance. He also showed the function generation problem to be convertible to a rigid-body guidance problem by inversion about the input or output links.

The advancing computer technology of the late 1950's gave rise to the first numerically-based attempts to optimize planar mechanisms. Freudenstein (20) devised an iterative scheme for respacing the precision points in order to find the linkage which best approximates a given function. Starting with the well-known Chebychev spacing of the precision points, a mechanism was synthesized. An analysis routine then evaluated the resulting structural error over the range of the desired function. Following this, a new "modified Chebychev" spacing of the precision points was created, and the analysis step repeated. The resulting errors of the two trials were then compared, and, based on the result, a new spacing could be projected.

Roth, Freudenstein and Sandor (21) used a digital computer to synthesize planar four-bar path generating linkages with optimum transmission characteristics. This was done by synthesizing linkages to satisfy four precision conditions in addition to requiring the maximum and minimum

values of the transmission angle to deviate from 90° by the same numerical amount. Synthesized mechanisms were then compared by means of a "quality index" formula and iterations were performed until the mechanism with the best value of the index could be found.

Until the work of Chi-Yeh (22) was published in 1966, synthesis methods invariably required the exact satisfaction of some number (usually 3,4 or 5) of precision conditions. Chi-Yeh's novel approach was to minimize the structural error in the least-squares sense at a much larger set of prescribed positions. Chi-Yeh's method involved taking the partial derivatives of the objective function with respect to the design variables (the linkage dimensions), and setting these equal to zero. This resulted in a system of nonlinear equations which were then solved numerically.

Nonlinear programming was first introduced to mechanism optimization by Fox and Willmert (23) in 1967. Their design objective was to synthesize a four-bar linkage whose coupler point would generate, as closely as possible, a given curve, and whose crank rotations would be as close as possible to the desired values. Constraints were imposed which limited forces and torques within the linkage, restricted fixed pivot location, limited link length ratios, assured the desired Grashof type and required the output positions to be in a

specified order. Also, the possibility of limiting velocities and accelerations below a certain value was discussed. This paper is remarkable in that it introduced nonlinear programming to kinematic synthesis in addition to developing a number of the constraint conditions which are still in common use today. In fact, many of the more recent works in this area have been in an effort to find more efficient optimization algorithms; however, the objective function and constraint equations have been virtually the same as those suggested by Fox and Willmert. Freudenstein, in his discussion of the paper (23), described the nonlinear programming method as a "natural tool" in the area of kinematic synthesis.

Closely following, but apparently independent from, the work of Fox and Willmert, was a paper by Tomas (24) which also treated linkage synthesis as a nonlinear optimization problem. Noteworthy about the work of Tomas was the first treatment of the function generation problem by nonlinear programming, and the first published example of optimization with respect to dynamic properties of the mechanism.

A different tack to the planar linkage optimization problem was presented by Garrett and Hall (25). They chose to generate, by means of a digital computer, a set of random four-bar linkages. Each of these linkages was then analyzed to find the one which best suited the

design requirements. Following this, an expanded set of random linkages was generated in the neighborhood of this "best" design, and repeating the analysis step, a new best linkage was selected. This refinement process was repeated until the desired accuracy was obtained, or until the method converged. Although it is usually inefficient, this method is quite simple and is almost guaranteed to converge. Furthermore, it may have a better chance of finding the global optimum (26), although no proof of this is known to exist.

Eschenbach and Tesar (27) contributed to the theory of planar linkage optimization on two important fronts. First, they treated the formerly unsolved problem of optimum design of a coplanar-motion generating four-bar linkage. Second, they were the first to recognize the simplification which resulted from using precision position equality constraints to reduce the number of free-choice variables.

Least-squares error synthesis of planar four-link mechanisms was a popular subject during the 1960's, and attracted the interest of such authors as Lewis and Gyory (28), Levitskii and Sarkisian (29) and McLarnan (30) among others. Although none of these papers directly employ nonlinear programming methods, they do point out the importance given to the problem of minimizing structural error. Among these works, only the paper of

Lewis and Gyory (28) considers the need to satisfy additional constraints. Here, a test is performed to be sure the synthesized mechanism is of the crank-and-rocker type. Structural error minimization was also the objective of Sandor and Wilt (31) in their novel paper on the optimal synthesis of a geared four-link mechanism.

Tomas (32) discounts the importance of minimizing structural error in many practical problems, and again presents the idea of optimization with respect to the dynamic properties of the mechanism. Benedict and Tesar (33) reinforce this concept by demonstrating the design of a complex stamping and indexing machine with optimal torque balance. Improved dynamic properties and balancing of planar mechanisms were also the objective of a number of other papers, including those by Berkof and Lowen (34,35), Huang, Sebesta and Soni (36), Sadler and Mayne (37) and Elliot and Tesar (38). Kaufman and Sandor (39) developed a complete force balancing method for spatial mechanisms.

Variations of the planar mechanism optimization problems already mentioned have been the subject of numerous other papers. Most of these papers attempt to improve the efficiency of the optimization process, either by using slightly different problem formulations, or by using one of the recently improved optimization

methods. These include the works of Golinski (40), Alizade, Novruzbekov and Sandor (41), Rose and Sandor (42), Savage and Suchora (43), Nolle and Hunt (44), Bagci and Brosfield (45) and Bagci and Lee (46).

As work in the area of mechanism optimization progressed, the importance of good problem formulation combined with an efficient numerical optimization method became apparent. Kinematicians began to modify existing optimization methods, or, in some cases, create new methods, which would be tailored to the peculiarities of mechanism design. Huang (47) developed a mechanism optimization scheme based on sensitivity coefficients. Lee and Freudenstein (48) introduced hueristic combinatorial optimization in the kinematic design of mechanisms. A hueristic method for sorting mechanism variables into independent groups was proposed by Datseris and Freudenstein (49). Sutherland and Siddall (50) presented a method for optimization using what they called "inverse utilities" as a basis for comparing the undesireable characteristics of a mechanism. Spitznagel (51) and Spitznagel and Tesar (52) developed a very effective technique for optimizing planar mechanisms based on Burmester synthesis followed by sequential filtering.

Other interesting and important problems concerning planar mechanisms were also being addressed in the literature. Optimal synthesis of six-link and other

multi-loop mechanisms was studied by Chen and Dalsania (53), Prasad and Bagci (54), Sallam and Lindholm (55), Mariante and Willmert (56) and Spitznagel (57). Dhande and Chakraborty (58) were the first to use stochastic methods to optimize mechanisms considering the effects of tolerances and clearances. Sevak and McLarnan (59) considered the problem of optimizing mechanisms with flexible links. Huey and Dixon (60) studied the optimization of cam-modulated linkages for path and function generation. Kramer and Sandor (61) and Kramer (62) developed an optimization technique based on what they termed "selective precision," wherein structural errors of the mechanism are held within a prescribed tolerance at each of the prescribed positions. Sutherland (63) presented a method for designing four-bar linkages where some of the prescribed positions are satisfied exactly and the remaining positions are approximated in the least-squares error sense. Chouby and Rao (64) suggest a direct method for minimizing the structural error together with the mechanical error due to manufacturing tolerances on the link dimensions.

Application of numerically based optimization methods to the design of spherical and spatial mechanisms became a topic of interest in the early 1970's. Stridher and Torfason (65) minimized the structural error in a spherical four-link path-generating mechanism. Bagci

and Parekh (66) demonstrated the optimal design of spherical four-bar and six-bar linkages. Bagci (67) designed spherical four-link mechanisms to have optimal transmission properties. Rao and Ambekar (68) considered the design of spherical four-link function-generating mechanisms with minimum structural error subject to link length and transmission angle constraints.

Early efforts to optimize spatial mechanisms were concerned mainly with either minimizing structural error or maximizing force or load transmission, or some combination of the two. Research in this category includes the work of Sutherland and Siddall (50), Hamid and Soni (69) and Shoup, Steffen and Weatherford (70). Gupta (71) demonstrated the synthesis of RSSR and RSRC spatial linkages with minimum structural error, subject to branching, mobility and transmission constraints. Suh and Mechlenburg (72) synthesized path-generating spatial mechanisms with minimum structural error in the least-squares sense. In a unique work, Bagci (73) optimized screw-generating spatial mechanisms. Alizade, Rao and Sandor (74,75) demonstrated the synthesis of spatial function-generating mechanisms with minimum error subject to transmission angle constraints. Suh and Radcliffe (76) used nonlinear programming methods for the precision position synthesis of spatial mechanisms. Although closed-form solutions were possible for some of the

mechanisms synthesized by Suh and Radcliffe, their work demonstrates that it is possible to circumvent much of the labor-intensive manipulation of algebraic equations if one is willing to pay for more computer time. Bagci and Falconer (77) optimized the transmission characteristics of RSSR and RSSP function-generating spatial mechanisms. Söylemez and Freudenstein (78) found the optimum dimensions of an RSSR function-generating mechanism when the extreme positions of the input and output links are given. Karelin (79) obtained formulas for determining the optimum link sizes and slider offset for the RSSP mechanism given the required stroke. His design was subject to constraints on the pressure angle.

A number of other papers which do not fall directly in the category of optimization are, nevertheless, important to the present study. These can be classified into two groups: papers dealing with determining the quality of an existing mechanism and papers dealing with precision position synthesis of spatial mechanisms.

The first group is important to this study because all mechanism optimization techniques are based on quality comparisons. Perhaps the simplest and best known quality criterion is the Grashof mobility test (80) for planar four-bar linkages. Filemon (81) extended the usefulness of this test by determining the regions

of mobility along Burmester's centerpoint curve. Jenkins, Crossley and Hunt (82) studied the gross motion attributes of certain spatial mechanisms based on the intersections of surfaces generated by points within the mechanism. Duffy and Gilmartin (83,84,85) generalized Grashof's work by determining the limit positions and the mobility of four-link spatial and spherical mechanisms using the laws of spatial and spherical triangles. Gupta and Radcliffe (86) used geometric methods and design charts to determine the mobility of planar and spatial mechanisms. Waldron and Stevensen (87) worked on the development and application of conditions on branching, mobility and order of positions in planar four-bar linkages. Strong and Waldron (88) used joint displacements to determine the mobility regions of four-bar linkages. Sutherland (89) developed an index for determining the quality of force and motion transmission of planar and spatial mechanisms. Gupta (90,91) developed theories for synthesizing crank-type planar four-bar linkages with transmission angle control. Gupta and Tinubu (92) proposed a theory for synthesizing planar and spatial bimodal function generating mechanisms which were free from branching problems. Zhuang and Sandor (93,94) determined the branching condition for a variety of spatial mechanisms containing spheric joints.

Several early works dealing with the precision position synthesis of spherical and spatial mechanisms have already been discussed. Although the subject is too broad to fully cover here, a few additional references are cited to give the reader an idea of the methods currently available.

Dimentberg (95) introduced the screw method to the study of lower-pair spatial mechanisms. This elegant and powerful tool provided a concise method for formulating and solving spatial mechanism problems. Often the screw method provided closed-form solutions to problems which otherwise require numerical solutions when formulated using vector or matrix methods. Sandor (96) and Sandor and Bisshopp (97) introduced methods of dual number quaternions and stretch-rotation tensors to find the loop-closure equations of spatial mechanisms. Beran (98) used dual complex numbers to synthesize the RCCC mechanism for multiply separated positions. Tsai and Roth (99) used screw triangle geometry to synthesize open-loop kinematic chains. Kohli and Soni (100) developed spatial mechanism synthesis procedures based on pair geometry constraints and successive screw displacements. Suh (101) employed 4x4 matrices in the synthesis of spatial mechanisms.

Recently, Sandor, Kohli, Reinholtz and Ghosal (102) developed an analytical closed-form method for the

synthesis of a spatial motion-generating mechanism for three prescribed positions based on vector geometry. Sandor, Kohli, Zhuang and Reinholtz (103) extended this work to four prescribed precision positions; their solution involves finding the simultaneous solutions of two cubic equations in three unknown variables. The value of one of the variables is assumed arbitrarily in solving the equations.

While fairly extensive, this literature review is by no means complete, and the reader is encouraged to peruse the excellent review articles by Fox and Gupta (104) and Root and Ragsdell (105).

1.4 Conclusions of the Literature Review

The foregoing literature review reveals a number of important points:

(1) In recent years, optimization theory has been widely recognized as an important and natural tool to be used in the design of mechanisms.

(2) No single mechanism optimization procedure has received widespread acceptance. This is partially due to the newness of the optimization methods themselves, and partially due to the wide variety of problems

which occur in mechanism design.

(3) In a majority of the papers reviewed dealing with optimization, the objective function was in some way a measure of the structural error, although no reason for this choice was generally given. It is believed that this approach arose as a "natural extension" of the methods of precision position synthesis.

(4) The authors who chose to optimize with respect to properties other than structural error most often incorporated either transmission quality or some dynamic property of the mechanism in the objective function. Quite often in these works equality constraints were used to force the solution mechanism to pass through a small number (usually 3 or 4) of precision positions.

(5) Very few designers have attempted to devise a general philosophy or strategy for mechanism optimization. As a result, many of the methods developed have limited application or are relatively

inefficient or both.

(6) The literature contains a number of very fine examples of planar mechanism optimization. Also, several useful techniques have been developed for the synthesis and analysis of spatial mechanisms. However, the extension of these techniques to spatial mechanism optimization has been quite limited.

CHAPTER 2

OPTIMIZATION THEORY

2.1 Introduction to Optimization

Optimization has previously been defined as the act of seeking the best result under a given set of circumstances. Finding the best result ultimately means minimizing something (such as the required effort), or maximizing something (such as the desired benefit). In order to have a solvable optimization problem, the desired benefit or the required effort must be expressible as a function of a set of variables over which the designer has control. These variables are called the design variables. Limits on the values of design variables may result from such things as limited material supplies or limited production capabilities. In either case, these limiting factors are called constraints, and they, too, must be expressible as functions of the design variables.

From figure 2.1 it can be seen that the maximum value of a function, $f(x)$, corresponds to the minimum value of the negative of the function, $-f(x)$. Therefore,

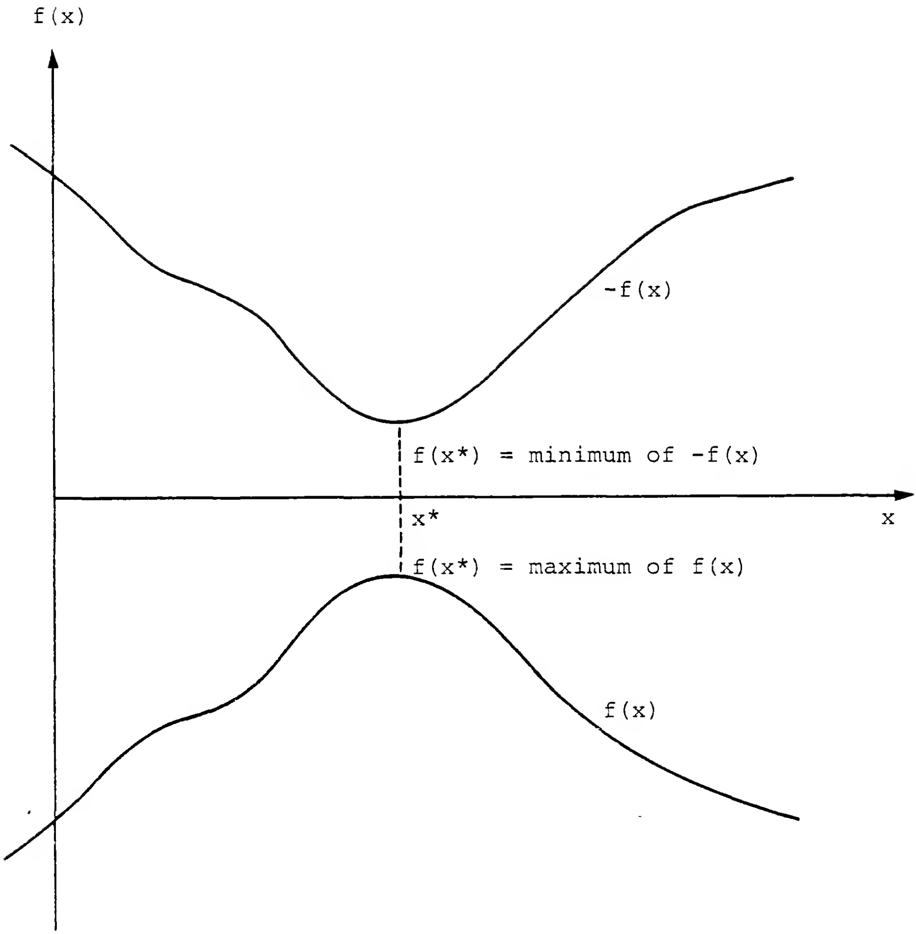

figure 2.1 Example showing the maximum of f(x) to be the same as the minimum of -f(x)

no generality is lost by assuming the optimization problem to always be one of function minimization.

2.2 Formal Definition of the Optimization Problem

An optimization problem or a mathematical programming problem can be stated in the following general form (104,106):

$$\text{Find} \qquad \underset{\sim}{X} = \begin{Bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{Bmatrix} \tag{2.1}$$

which minimizes $f(\underset{\sim}{X},\underset{\sim}{\Theta})$ (where $\underset{\sim}{\Theta}$ is an independent set of input motion variables having a predetermined range), subject to inequality constraints

$$g_j(\underset{\sim}{X},\underset{\sim}{\Theta}) \leq 0, \qquad j = 1,2,\ldots,m \tag{2.2}$$

and equality constraints

$$\ell_j(\underset{\sim}{X},\underset{\sim}{\Theta}) = 0, \qquad j = m+1,\ m+2,\ \ldots,\ p \tag{2.3}$$

Here $\underset{\sim}{X}$ is called the design vector, and $f(\underset{\sim}{X},\underset{\sim}{\Theta})$ is called the objective function. The design vector, $\underset{\sim}{X}$, is composed of the design variables x_1, x_2, ..., x_n, which may be written in transposed 1xn matrix form as $\{x_1,x_2,\ldots,x_n\}^T$, where the superscript T denotes the transpose. The vector $\underset{\sim}{\Theta} = \{\theta_1,\theta_2,\ldots,\theta_q\}^T$ of a set of independent parameters

may typically comprise quantities such as time or position. Most types of optimization problems do not contain this independent parameter vector, and, hence, most textbooks do not include it in defining the standard optimization problem. However, in mechanism design, the independent parameter(s) representing the mechanism input(s) often appear in the design equations.

When the vector $\underset{\sim}{\theta}$ is present in the objective function or in any of the constraint equations, the problem will be called a parametric programming problem. This type of problem is generally much more difficult to solve than nonparametric problems, because for each new design vector, $\underset{\sim}{X}$, that is tried, the objective function and the constraints must be evaluated at every possible value of the independent input parameter(s). In this dissertation, a great deal of emphasis will be placed on avoiding the need to involve these independent parameters, while assuring optimal performance throughout their range.

Finally, it should be pointed out that the components of the design vector, $\underset{\sim}{X}$, may also be functions of independent parameters. This occurs, for example, when link shapes are considered variables in the design of a mechanism which is to have maximum stiffness. This type of optimization problem is known as a dynamic programming problem. No dynamic programming problems have been included among the examples of this dissertation.

2.3 The Mechanism Optimization Problem

To put the preceding definition of the optimization problem in a clearer light, it will now be cast in the framework of application to the design synthesis of a simple mechanism.

Suppose it is desired to design a function-generating planar four-bar linkage for a number of arbitrarily prescribed positions greater than five. Since, in this case, five is the maximum number of positions that can be satisfied exactly (unless input and output scale factors are made components of the design vector), the best that can be hoped for is to minimize the structural error at the prescribed positions. A logical and popular approach to this problem has been to minimize the sum of the squares of the resulting structural errors at the prescribed positions. Thus, if the output angle, ϕ_i, is some known function, F, of the input angle, θ_i, at each of the n prescribed positions, then

$$\phi_i = F(\theta_i), \qquad i = 1,2,...n, \tag{2.5}$$

and the objective function (O.F.) to be minimized is

$$\text{O.F.} = \sum_{i=1}^{n} \{F(\theta_i) - G(\theta_i)\}^2 \tag{2.6}$$

where $G(\theta_i)$, $i = 1,2,...,n$ are the output positions actually generated by the mechanism. The mechanism to be optimized is shown in figure 2.2.

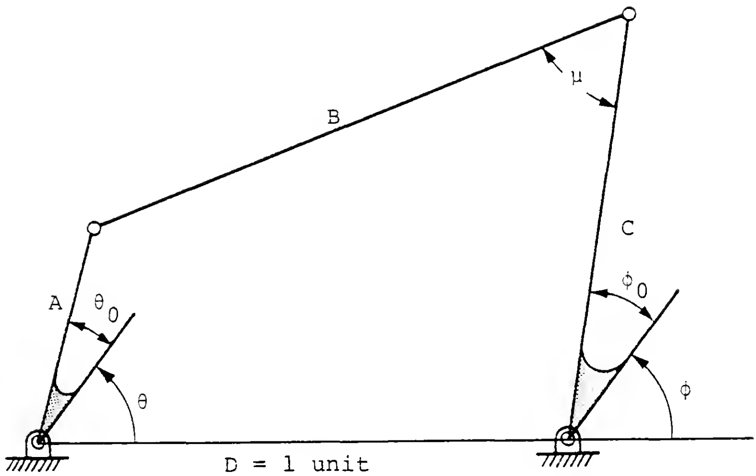

figure 2.2 The function-generating planar four-bar linkage

The design vector consists of the design variables which are the link lengths A,B,C,D and the starting angles θ_0 and ϕ_0. For a given value of $\theta=\theta_i$, the values of these variables determine the values of $F(\theta_i)$ and $G(\theta_i)$. The values of the variables A,B,C,D,θ_0 and ϕ_0 are to be found which minimize the function 2.6.

The reader familiar with kinematics will know that the relative, rather than the absolute, proportions of this mechanism determine the input/output functional relationship. Therefore, D, the length of the grounded link, can be set equal to unity without loss of generality. The design vector now becomes

$$\underset{\sim}{X} = \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{Bmatrix} = \begin{Bmatrix} A \\ B \\ C \\ \theta_0 \\ \phi_0 \end{Bmatrix} = \{A,B,C,\theta_0,\phi_0\}^T \qquad (2.7)$$

While finding the value of $\underset{\sim}{X}$ which minimizes the function 2.6 may be quite an interesting problem in itself, in practice there are usually a number of other requirements the linkage must satisfy. For example, the transmission angle[1], labeled μ in figure 2.2, must be

---

[1]The transmission angle in a four-bar mechanism is defined as the acute angle between the coupler link and the output link.

held reasonably close to 90° to ensure effective conversion of the force in the coupler to torque about the output crank pin. In practice, a value of μ less than about 30 will generally be unacceptable. This requirement can be expressed in the form of an inequality constraint equation

$$\mu \geq 30^\circ \qquad \text{or} \qquad \mu - 30^\circ \geq 0 \tag{2.8}$$

However, this is not an acceptable form of the equation because the transmission angle, μ, is not a design variable or an independent parameter. However, μ can be expressed in terms of the design variables and the independent parameter θ as follows (107,pp319):

$$\mu = \arccos \left| \frac{A^2 - B^2 - C^2 + 1 - 2A\cos(\theta + \theta_0)}{2BC} \right| \tag{2.9}$$

Upon substituting this result into the transmission angle constraint, equation 2.8 becomes

$$\arccos \left| \frac{A^2 - B^2 - C^2 + 1 - 2A\cos(\theta + \theta_0)}{2BC} \right| - 30^\circ \geq 0 \tag{2.10}$$

Unfortunately, this is a parametric constraint since it contains the independent parameter θ. It therefore seems that equation 2.10 must be evaluated at every possible value of θ, from $\theta=0^\circ$ to $\theta=360^\circ$, for every set of design variables tried, to be sure the constraint is satisfied.

Fortunately, this is not usually necessary because, as shown by Roth, Freudenstein and Sandor (21), the transmission angle, μ, is a maximum when $\theta + \theta_0 = 180°$ and a minimum when $\theta + \theta_0 = 0°$, provided these positions are real, i.e., if the linkage closes in these positions of the input link. This demonstrates an extremely important concept: the need for parametric constraints can sometimes be eliminated by determining the critical values of the independent parameter.

Nonparametric constraints can result from a number of requirements. For example, it is usually desirable to limit the ratio of link lengths within a mechanism; otherwise solutions will result with extremely long coupler links and short cranks (for practical purposes, these become slider-crank mechanisms). To eliminate these unwanted solutions, the following constraints are specified:

$$A_\ell \leq A \leq A_u$$

$$B_\ell \leq B \leq B_u$$

$$C_\ell \leq C \leq C_u \tag{2.11}$$

or, expanding these into the standard form of separate inequalities,

$$A_\ell - A \leq 0 \qquad A - A_u \leq 0$$

$$B_\ell - B \leq 0 \qquad B - B_u \leq 0$$

$$C_\ell - C \leq 0 \qquad C - C_u \leq 0 \tag{2.12}$$

where, again, the length of the fixed link, D, is arbitrarily set equal to unity, and the subscripts ℓ and u refer to the lower and upper limits of the link length.

Equality constraints in linkage synthesis problems most often result from precision position requirements. Referring to the notation used in equation 2.6, suppose that, at the three positions q,r and s, the actual mechanism output is required to be the same as the specified output, then

$$F(\theta_i) = G(\theta_i), \qquad i = q,r,s \tag{2.13}$$

or

$$F(\theta_i) - G(\theta_i) = 0, \qquad i = q,r,s \tag{2.14}$$

are the needed equality constraints. Positions q,r and s may now be removed from consideration in equation 2.6, although this is not mandatory since they do not contribute to the value of the objective function. An important point which will be studied in detail in section 2.3.2 is that equality constraints can very often be used to eliminate variables from consideration, without any loss of generality in the problem formulation.

Summarizing the four-bar linkage optimization problem at hand,

find

$$\underset{\sim}{X} = \{A,B,C,\theta_0,\phi_0\}^T \tag{2.15}$$

which minimizes

$$\sum_i \{F(\theta_i) - G(\theta_i)\}^2,$$

$$i = 1,2,\ldots,q,\ldots,r,\ldots,s,\ldots,n \qquad (2.16)$$

subject to

$$\arccos\left|\frac{A^2 - B^2 - C^2 + 1 + 2A}{2BC}\right| - 30° \geq 0 \qquad (2.17)$$

$$\arccos\left|\frac{A^2 - B^2 - C^2 + 1 - 2A}{2BC}\right| - 30° \geq 0 \qquad (2.18)$$

$$\begin{aligned} A_\ell - A &\leq 0 \\ A - A_u &\leq 0 \\ B_\ell - B &\leq 0 \\ B - B_u &\leq 0 \\ C_\ell - C &\leq 0 \\ C - C_u &\leq 0 \end{aligned} \qquad (2.19)$$

and

$$F(\theta_i) - G(\theta_i) = 0, \qquad i = q,r,s \qquad (2.20)$$

Notice that the critical values $(\theta + \theta_0) = 0°$ and $(\theta + \theta_0) = 180°$ have been substituted into constraint equation 2.10 to yield equation 2.17 and 2.18. Also, the positions at which the function must be exactly

satisfied, θ_i, i = q,r,s, have not been removed from consideration in the objective function.

A number of other constraints could be added to this problem. For example, it is often desirable to have a crank-rocker type linkage, which can be driven from a continuously rotating prime mover. The inequality constraint which ensures this type of mechanism is derived from the so-called Grashof criterion. Other constraint conditions will be discussed in Chapter Three. The present problem is sufficiently general for use in discussing the various methods available for solving the general nonlinear optimization problem which typically results from mechanism optimization; these solution methods are presented in the following section.

2.4 Solving the Mechanism Optimization Problem

Optimization problems may be either linear or non-linear. Nonlinear problems occur when the objective function or any of the constraint equations are nonlinear in any of the design variables. It will not surprise the experienced kinematician to know that almost all mechanism optimization problems fall into this category. Even judging from the simple example of the previous section, it becomes evident that these types of problems are generally nonlinear, and usually involve a large number of constraints. For this reason, the

remainder of this section will deal exclusively with methods for solving constrained, nonlinear optimization problems.

2.4.1 Constrained Nonlinear Optimization

A number of methods are currently available for solving constrained nonlinear optimization problems. While some of these methods are more widely used than others, no single method is best suited to solve every type of problem. In fact, even small changes in the way a problem is formulated can grossly alter the effectiveness of the optimization procedure being used. This explains, in part, the large number of past approaches which have been taken in mechanism optimization, and the claims, often conflicting or confusing, about the efficiency and the effectiveness of these approaches (104,105).

Although it is possible to solve some constrained nonlinear optimization problems using the classical techniques of variational calculus, the complexity of most mechanism optimization problems renders this approach impractical. Therefore, the remainder of this section will be devoted to the iterative solution methods known as constrained nonlinear mathematical programming, or simply constrained nonlinear programming.

Constrained nonlinear programming techniques can be divided into two distinct groups: direct methods and indirect methods. Table 2.1 shows the various techniques that come under these headings (106).

Table 2.1

Constrained Nonlinear Programming Techniques

Direct Methods	Indirect Methods
1) Heuristic search	1) Transformation of variables
2) Constraint approximation	2) Penalty functions
3) Feasible directions	

Classification is based on the manner in which the constraints are handled. The direct methods deal with the constraints explicitly, whereas the indirect methods first transform the constrained problem into an unconstrained problem, and then solve this new easier problem using one of the unconstrained nonlinear programming methods.

All of the methods listed in Table 2.1 are potentially useful in mechanism optimization. However, it will not be practical to discuss each of these methods in detail. Since the indirect methods are used in working the examples of this dissertation, these will be discussed in greater detail. Complete details on

all of the methods listed are available in the references (106,108,109).

2.4.2 Indirect Constrained Nonlinear Programming Techniques

As can be seen from Table 2.1, two methods come under this heading: transformation of variables and penalty functions.

In the transformation of variables technique, the design variables are changed in such a way as to automatically ensure constraint satisfaction. There are two cases where this is possible: (1) when the constraints are simple, explicit functions of the decision variables, and (2) when equality constraints can be used to eliminate variables.

As an example of the first case, recall the first of the link length constraint equations 2.11 of section 2.2, repeated here as equation 2.21.

$$A_{\ell} \le A \le A_u \tag{2.21}$$

where A was the length of the input crank of the four-bar linkage of figure 2.1, and A_{ℓ} and A_u were the upper and lower limits placed on this length. These constraints can be automatically satisfied by transforming the variable A to the form

$$A = A_{\ell} + (A_u - A_{\ell})\sin^2 A^*, \tag{2.22}$$

where A^* is the new variable which can take on any numerical value. This technique is called change of variables. Notice the A will always be between A_ℓ and A_u for any value of A^* in equation 2.22.

While this approach seems quite promising at first, experience has shown that, unless all the constraints can be transformed in this way, it is probably better not to use the transform at all (102). This is because substitution of the right-hand side of equation 2.22 into the objective function may distort it to the point where it is more difficult to minimize than the original function, when other constraints are present. As a result, this approach appears to be impractical for most mechanism optimization problems, although the author believes further study in this area is warranted.

The second case for which transformation of variables is sometimes possible occurs when equality constraints are present. For example, in section 2.2, equation 2.13 expresses a set of three precision position requirements for the four-bar linkage of figure 2.1

$$F(\theta_i) = G(\theta_i), \qquad i = q,r,s \tag{2.23}$$

where the functions $F(\theta_i)$ and $G(\theta_i)$ express the desired and the generated output angular positions of the mechanism, respectively, at a given input position θ_i. Recall from equation 2.5 that $F(\theta_i) = \phi_i$ is a known, prescribed

function. Equation 2.23 may be rewritten in the form of the well-known Freudenstein equation (1), which gives the input/output angular relationship of the four-bar function generator in terms of the linkage dimensions

$$K_1\cos(\theta_i' + \theta_0) - K_2\cos(\phi_i + \phi_0) + K_3 =$$

$$\cos(\theta_i + \theta_0 - \phi_i - \phi_0) \qquad (2.24)$$

where

$$K_1 = \frac{1}{C}; \qquad K_2 = \frac{1}{A}; \qquad K_3 = \frac{A^2 - B^2 + C^2 + 1}{2AC} \qquad (2.25)$$

and θ_0 and ϕ_0 are the starting angles (see figure 2.2). Equation 2.24 is linear in the coefficients K_1,K_2 and K_3, and can be written three times corresponding to the three values of i (i.e. i = q,r and s). It is therefore a relatively easy matter to solve for the values of K_1, K_2 and K_3 in terms of the angles θ_i, ϕ_i, i = q,r,s, and θ_0, ϕ_0 (101). With K_1, K_2 and K_3 known, the link lengths may be determined from the relations

$$A = \frac{1}{K_2}; \quad C = \frac{1}{K_1}; \qquad B = \{A^2 + C^2 + 1 - 2ACK_3\}^{\frac{1}{2}} \qquad (2.26)$$

Notice that the only design variables remaining in the right-hand sides of equations 2.26 are the starting angles

θ_0 and ϕ_0. Anywhere A,B and C appear in the remaining constraint equations or in the objective function they may be replaced by the equivalent expressions given in equation 2.26. This technique is called elimination of variables.

As in the previously-described change of variables technique, the objective function and the remaining constraints will undoubtedly be distorted when the non-linear expressions for A,B and C of equation 2.26 are substituted into them. However, experience has shown that it is generally beneficial, in this case, to make the substitution (26,51,52,63,106). Obviously, the greater the degree of nonlinearity of the expressions used to eliminate variables, the less beneficial this type of substitution becomes.

It has already been noted that the role of the indirect methods of constrained nonlinear programming is to transform the constrained problem into an equivalent unconstrained problem. Occasionally, the transformation of variables method can accomplish this goal by itself. More often, however, some of the constraint equations are too complex for this, and the so-called penalty function approach must be employed.

To demonstrate the penalty function approach, consider the following simple problem, where the parametric vector, $\underset{\sim}{\theta}$, has been omitted for clarity.

Find $\underset{\sim}{X}$ which minimizes $f(\underset{\sim}{X})$

subject to

$$g_j(\underset{\sim}{X}) \leq 0, \qquad j = 1,2,\ldots,m \tag{2.27}$$

and

$$\ell_j(\underset{\sim}{X}) = 0, \qquad j = m+1,m+2,\ldots,p$$

This constrained problem is converted into an unconstrained problem by constructing a new function to be minimized of the form

$$U = U(\underset{\sim}{X},r) = f(\underset{\sim}{X}) + r\sum_{j=1}^{m} G_j\{g_j(\underset{\sim}{X})\}$$

$$+ r\sum_{j=m+1}^{p} L_j\{\ell_j(\underset{\sim}{X})\} \tag{2.28}$$

where $G_j\{g_j(\underset{\sim}{X})\}$ and $L_j\{\ell_j(\underset{\sim}{X})\}$ are functions of the constraint functions $g_j(\underset{\sim}{X})$ and $\ell_j(\underset{\sim}{X})$, respectively and r is a positive constant called the penalty parameter. The solution of the unconstrained problem of equation 2.28 can be made to converge to the solution of the original problem of equation 2.27 by repeating the minimization process for a progressively larger series of values of the penalty parameter, r. For this reason, the penalty function methods are often referred to as "sequential unconstrained minimization techniques" or simply SUMT.

Two categories of penalty function methods exist, namely, interior methods and exterior methods. The interior methods must be supplied with a feasible starting vector, $\tilde{X}_1$ (i.e. $g_j(\tilde{X}_1) \leq 0$ for all j). As the parameter r is varied over successive minimizations, the solution of the unconstrained problem converges to the solution of the constrained problem, always remaining within the feasible region. Since the search is conducted within the feasible region, these are called interior methods. The exterior methods do not require a feasible starting vector, and generally converge to the constrained minimum from outside the feasible region, hence the term exterior. The exterior methods have been judged to be generally superior to the interior methods (109); therefore, these will be reviewed in greater detail.

A typical exterior penalty function form of equation 2.28 is

$$U(\tilde{X},r) = f(\tilde{X}) + r \sum_{j=1}^{m} \left\langle g_j(\tilde{X}) \right\rangle^q + r \sum_{j=m+1}^{p} \{\ell_j(\tilde{X})\}^2 \qquad (2.29)$$

where, again, r is a positive penalty parameter, q is a constant greater than one, and the singularity function $\left\langle g_j(\tilde{X}) \right\rangle$ is defined by

$$\left\langle g_j(\underset{\sim}{X}) \right\rangle = \begin{cases} g_j(\underset{\sim}{X}) & \text{if} \quad g_j(\underset{\sim}{X}) > 0 \\ 0 & \text{if} \quad g_j(\underset{\sim}{X}) \leq 0 \end{cases} \tag{2.30}$$

It can be seen that the effect of this formulation is to assess an increasingly severe penalty on the value of $U(\underset{\sim}{X},r)$ as the constraints become violated by larger amounts. The most successful way to find the true constrained minimum of the original function has generally been to minimize equation 2.23 using a small value for the parameter r for the first minimization. Subsequent minimizations use successively larger values of this parameter, until the solution is essentially forced to converge in the feasible region. This is necessary because, if r remained small, very small positive values of $g_j(\underset{\sim}{X})$, even though infeasible, would not contribute much of a penalty to $U(\underset{\sim}{X},r)$ and the solution might remain infeasible. On the other hand, if r were made initially very large, the contribution of $f(\underset{\sim}{X})$ would be negligible, and the solution may not converge to the minimum $f(\underset{\sim}{X})$ within the constrained region. In other words, the effects should be balanced, so that the solution is urged toward the minimum of $f(\underset{\sim}{X})$ at the same time it is being forced toward the feasible region. The simple example which follows will help to demonstrate some of these concepts.

Find $\underset{\sim}{X} = \{x_1\}$ which minimizes $f(\underset{\sim}{X}) = \frac{1}{2}x_1$ subject to the constraint

$$3 - x_1 \leq 0 \tag{2.31}$$

The objective function $f(\underset{\sim}{X})$ and the constraint boundary are plotted in figure 2.3. The constrained minimum is clearly at $x_1 = 3$. Now construct the exterior penalty function

$$U(\underset{\sim}{X},r) = \frac{1}{2}x_1 + r\left\langle 3 - x_1 \right\rangle^2 \tag{2.32}$$

Table 2.2 gives various values of $U(\underset{\sim}{X},r)$ versus x_1 for several values of r.

The resulting curves are plotted in figure 2.3. It is clear from this figure that, as r tends toward infinity, the solution to the unconstrained objective function of equation 2.32 will approach the solution to the constrained problem of equation 2.31. It may be noted that, in this case, the solution will reach the feasible region only in the limit as r approaches infinity. This usually is not troublesome for practical problems because the constraints are rarely known exactly, and some allowance must be made for errors when formulating them.

The preceding discussion focused on the use of penalty function methods and change of variable techniques to transform constrained optimization problems into

Table 2.2

Example showing the effects of the penalty parameter, r.

$U(\underset{\sim}{X},r)$

x_1	r=0.25	r=0.5	r=1
0	2.25	4.50	9.00
0.25	2.02	3.90	7.69
0.50	1.81	3.38	6.50
0.75	1.64	2.91	5.44
1.00	1.50	2.50	4.50
1.25	1.39	2.16	3.69
1.50	1.31	1.86	2.75
1.75	1.27	1.66	2.44
2.00	$\boxed{1.25}$	1.50	2.00
2.25	1.27	1.40	1.69
2.50	1.31	$\boxed{1.38}$	1.50
2.75	1.39	1.41	$\boxed{1.43}$
3.00	1.50	1.50	1.50

Bold squares indicate the tabulated minimum of $U(\underset{\sim}{X},r)$.

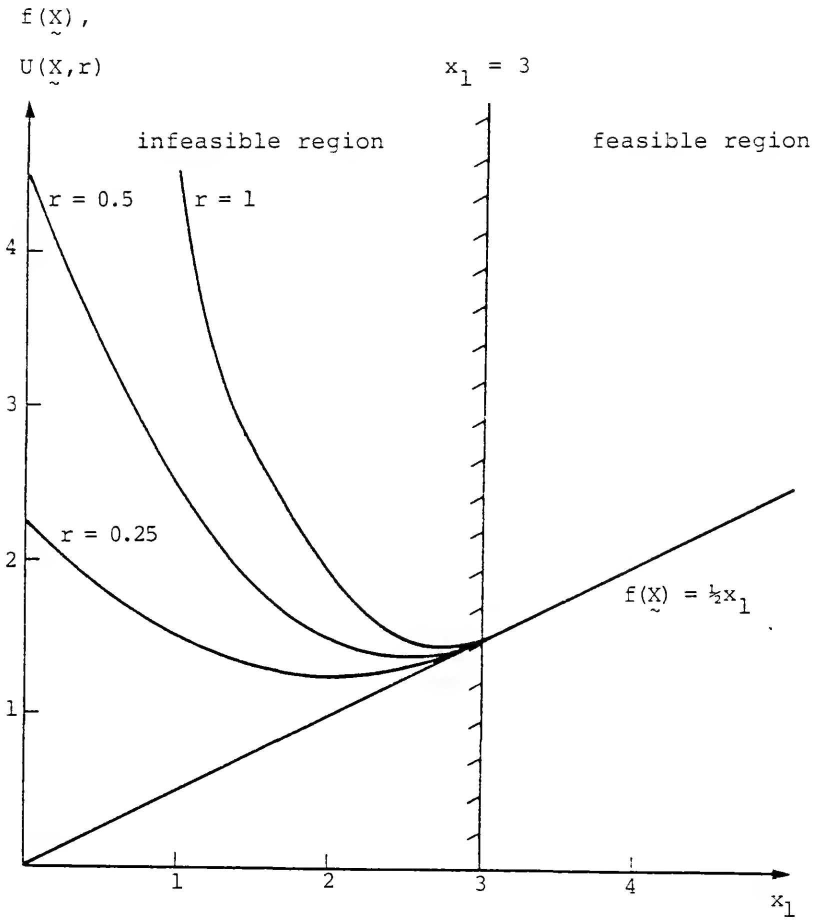

figure 2.3 Penalty function example

unconstrained ones. The question still remains of how these unconstrained problems may be solved; this is the topic of the next section.

2.4.3 Unconstrained Nonlinear Optimization

Even though the vast majority of nonlinear optimization problems involve constraints, most of the available nonlinear optimization techniques have been developed for solving unconstrained problems. This does not represent a serious limitation, however, because most of these methods can be extended to handle constrained problems, either by directly considering the constraints or by transformation to an unconstrained problem as discussed in the previous section.

Unconstrained minimization methods may be divided into two groups: direct search methods and descent (or gradient) methods. The gradient methods require either an analytical or a numerical derivative of the objective function with respect to the design variables, whereas the direct search methods do not. Some of the commonly available techniques in both groups are listed in Table 2.3 (106).

Of the methods listed in Table 2.3, the random search and the grid search are known to be quite inefficient. However, these methods tend to be reliable when minimizing discontinuous, sharply varying or

nondifferentiable functions. They may also be useful for finding feasible solutions to initiate some of the more efficient methods.

Table 2.3

Unconstrained Minimization Techniques

Direct Search Methods	Descent Methods
1) Random search	1) Steepest descent method
2) Grid search	2) Conjugate gradient method (Fletcher-Reeves)
3) Univariate search	3) Newton's method
4) Pattern search (Powell's method, Hooke and Jeeves' method)	4) Variable metric method (Davidon-Fletcher-Powell)
5) Method of rotating coordinates (Rosenbrock's method)	
6) Simplex method	

An excellent comparison of many of the numerical optimization methods commonly used to solve mechanical design problems was made by Eason and Fenton (109). They point out that the ideal computer code for design optimization should solve any problem conveniently and at moderate cost. No code tested by them fulfilled this requirement, but one method did stand out above the others, namely, the pattern search method of Hooke and Jeeves. A number of other important conclusions were

reached in this study; some of these are summarized below:

(1) Derivatives of the objective function are often difficult or impossible to calculate analytically for many mechanical design-type problems, and must be approximated by numerical methods, if needed.

(2) If derivatives must be calculated numerically, the direct methods (which do not require derivatives) are generally superior to the gradient methods.

(3) Automatic scaling of the design variables within the computer program generally increases the efficiency of an algorithm.

(4) The most general methods (those which could solve the most types of problems) were not necessarily slow, nor did they require the greatest amount of computer code to program.

(5) The cost of preparing a problem for computer solution may be greater than the execution cost. The algorithm should, therefore, be convenient to use.

(6) A computer package containing an assortment of optimization methods would generally be preferred to any single method. This

would allow cross-checking of results, and obviously would allow more types of problems to be solved than would any of the methods individually.

Development of a computer optimization package as described in conclusion number (6) above would be a major undertaking. Also, such a package would require a relatively large amount of computer storage. Still, this would be the preferred approach in terms of efficiency and generality. Since no such computer package is readily available, and since computer storage will often be an important consideration, it was decided to use the pattern search method of Hooke and Jeeves in working the examples of this dissertation. Later results will show this method to be quite effective in solving mechanism optimization problems. Good results using this method for optimizing planar mechanisms were also reported by Kramer and Sandor (61) and Kramer (62).

2.4.4 Hooke and Jeeves' Nonlinear Programming Method

Hooke and Jeeves' search method is a direct, sequential stepping technique consisting of alternating exploratory and pattern moves. The exploratory move seeks to determine the local behavior of the objective function, and the pattern move uses this information in

an attempt to "leapfrog" to an improved position. Figure 2.3 demonstrates this procedure for a two-dimensional problem, and the algorithm is outlined in detail below.

(1) Starting from an arbitrarily selected point X^0, located at $\underset{\sim}{X}^0 = \{\underset{\sim}{x}_1, \underset{\sim}{x}_2\}^T$, try an exploratory move by changing $\underset{\sim}{x}_1$ by a predetermined positive step $\Delta\underset{\sim}{x}_1$. Evaluate the objective function at this new point. If its value is improved, this step is retained, and becomes the new base point. If the value of the objective function is not improved, a negative step, $-\Delta\underset{\sim}{x}_1$, is taken and the objective function is reevaluated. If the move is successful, the point is retained as the new base point. If both steps fail, no move is made. At this point it is often beneficial to adjust the value of $\Delta\underset{\sim}{x}_1$, and save this information for future exploratory searches in the $\underset{\sim}{x}_1$ direction. A successful step move would suggest an increase in the value of $\Delta\underset{\sim}{x}_1$. If neither step were successful, the value of $\Delta\underset{\sim}{x}_1$ should be decreased. Starting from the best point found in the $\underset{\sim}{x}_1$ search (labeled X^1 in figure 2.3) a similar search is made in the $\underset{\sim}{x}_2$ direction. The best point found in this search is labeled X^2.

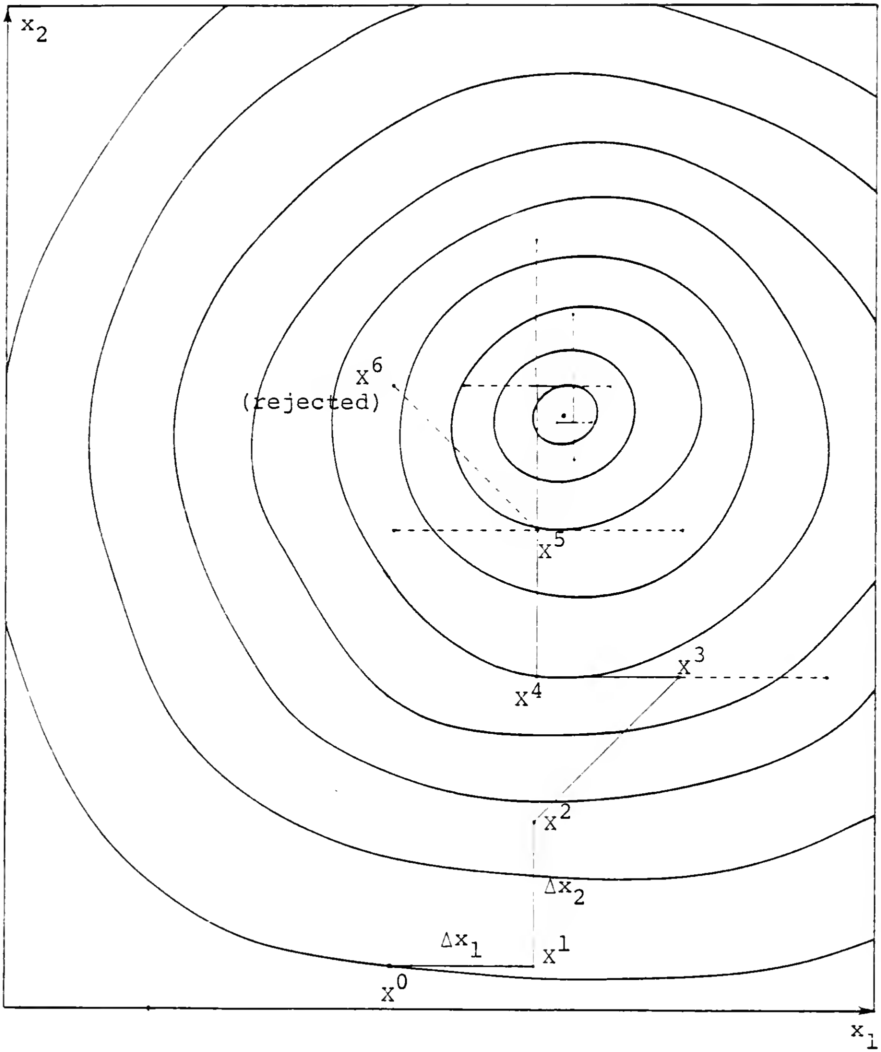

figure 2.3 The pattern search method of Hooke and Jeeves

(2) A pattern move is now attempted by repeating all the successful moves of the exploratory search from point X^2. After the initial search, this move may also include the previous exploratory moves and the previous pattern search.

(3) If the pattern move of part (2) is successful, it is retained as point X^3, and the exploratory search begins at this point. If the pattern move fails, the next exploratory search begins at point X^2.

(4) This process is repeated until the values of the Δx-s are below a certain preset limit. At this point the minimum is presumed to have been reached.

A flow chart of this procedure is shown in figure 2.4.

While simple in concept, the Hooke and Jeeves' method is extremely powerful. It should be cautioned, however, that none of the optimization methods discussed can discriminate between a local and a global minimum. Geven ten different starting points, the Hooke and Jeeves' method may converge to ten different local minima. As discussed at length in the next chapter, this will seldom represent a serious problem in mechanism optimization.

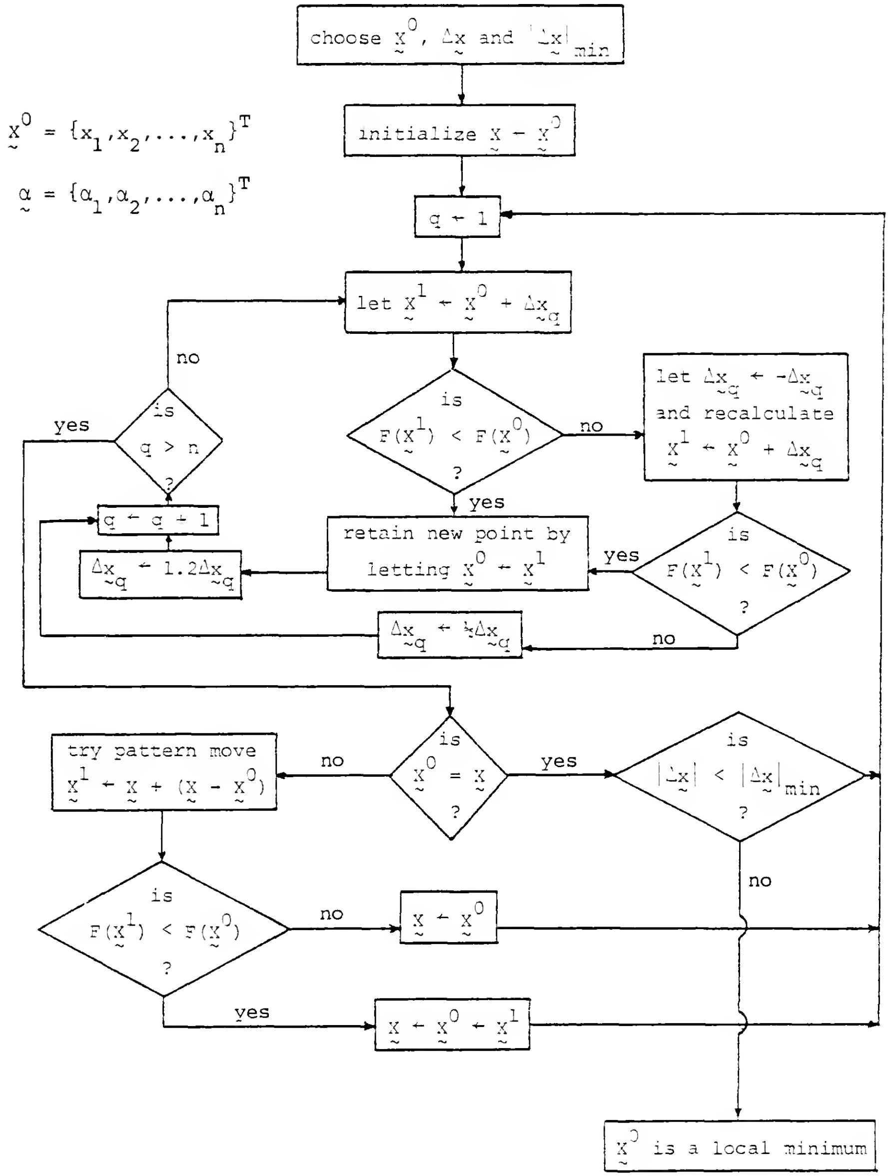

figure 2.4 Flow chart of the pattern search method of Hooke and Jeeves

CHAPTER 3

PHILOSOPHY OF MECHANISM OPTIMIZATION

3.1 The Need for a General Philosophy

Although theories for the kinematic synthesis and analysis of many types of spatial mechanisms are well developed and readily available, complete design theories for even the simplest four-link spatial mechanisms are not yet developed to the point where they are practical for use by the industrial machine designer. This "gap" exists because of the difficulty involved in selecting a single mechanism which satisfies all of the various real-world design requirements. This dissertation advocates optimization theory as the proper tool for bridging this gap. But while the application of optimization theory to planar mechanism design has risen at a seemingly exponential rate, there has been little application to spatial mechanism design. The author believes the reason is the relatively greater complexity and correspondingly greater number of parameters typically encountered in spatial mechanism design. For example, only six design variables are needed to specify the planar four-bar function-generating

mechanism discussed in section 2.2, whereas twenty-six such design variables may be needed to completely specify a spatial four-link RCCC motion-generating mechanism.[1] Even when a problem is rather poorly formulated, optimization can often be performed in a reasonable amount of time when dealing with six or ten design variables. However, the situation becomes decidedly different when dealing with twenty-six design variables. Since each variable may generally take on an infinite number of values, a twenty-six parameter problem represents on the order of ∞^{26} possible solutions! However, this does not mean that the application of optimization theory to problems involving a large number of design variables should be avoided. Quite the contrary, optimization theory offers the only real hope of solving such difficult problems. The point is that, as the number of design variables increases, so does the need for an efficient problem formulation coupled with an efficient optimization method and a means for reducing the number of parameters. Accordingly, this chapter attempts to define the goals of mechanism optimization, and to develop guidelines for obtaining these goals in

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[1]See the derivation of the RC and CC dyad synthesis equations in Chapter Four.

the most efficient manner; collectively, this will be called the philosophy of mechanism optimization.

3.2 Objectives and Constraints of Mechanism Optimization

There are typically a number of requirements which must be taken into account when designing a mechanism. Some of these are listed and described below.

(1) Motion specification. Most often a mechanism is required to generate (a) a functional relationship between input and output members; (b) a point path, sometimes coordinated with input motion, or (c) a rigid-body motion. Other motion requirements are possible, but these three have proven to be adequate for the vast majority of problems.

(2) Branch avoidance. Most mechanisms can be assembled in two or more distinct configurations while keeping the links connected in the same order. Each distinct configuration is called a branch. The mechanism will generally operate in one branch. Dissassembly is required to move it into a different branch. Branch avoidance implies the design of mechanisms such that the entire motion specification lies on one branch only.

(3) Order of positions. Precision positions must occur in the prescribed sequence and sense. For example, if the prescribed positions were given in the order 1,2,3,4, a mechanism generating these same positions, but in the order 1,3,2,4, would be unacceptable.[1]

(4) Grashof's condition. This refers to the relative rotatability of links within a mechanism. Often it is desirable to drive the input link of the mechanism from a continuously rotating source. Such an input is called a crank.

(5) Transmission characteristics. This refers to the effectiveness of the mechanism in transforming work at the input to work at the output. For some three- and four-link mechanisms, this can be expressed in terms of a transmission angle.

(6) Link-length ratio restrictions. Quite often, when trying to optimize the transmission characteristics of a mechanism, some

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[1]See Strong and Waldron (88) for a complete discussion of the order problem.

of the links within the mechanism become relatively much longer than others. This may cause manufacturing difficulties as well as other problems. To avoid this effect, limits may be placed on the allowed ratio of link lengths.

(7) Fixed-pivot location restrictions. Fixed pivots must be located so they do not interfere with other components. They must also be placed where they can be connected to the machine frame.

(8) Workspace restrictions. The operating space, or workspace, of the mechanism must often be restricted to avoid interference with other components.

(9) Dynamic and elasto-dynamic property restrictions. Velocities, accelerations, etc. and link deflections must remain within prescribed limits. This catagory also includes balancing requirements.

(10) Tolerance and clearance effect restrictions. The errors in mechanism output due to tolerances and clearances must be held within prescribed limits. Since these are not usually deterministic quantities, stochastic methods must often be employed.

Of course, all of these requirements will not apply to every problem. Quite often the designer can use his experience and judgment to eliminate several of these from consideration. For example, dynamic properties would probably not be a restrictive factor in a linkage designed for the purpose of guiding open an automobile hood. The reader can undoubtedly think of many other examples.

Classical mechanism design procedures generally begin by considering the motion specification. Typically, this results in a set of precision position requirements for function, path or rigid-body motion generation. These requirements are expressed as equality constraints which are then solved to yield the dimensions of one or more mechanisms which, at least mathematically, satisfy the precision positions. This procedure, commonly known as mechanism synthesis, is fascinating because of its unusual combination of geometric and mathematical complexities coupled with a concrete physical phenomenon. Unfortunately, this fascination has sometimes led to the feeling in academics that precision position synthesis is tantamount to mechanism design. This is not true, since each of the previously discussed requirements, and perhaps more, must be considered when designing a mechanism.

Precision position synthesis is so routinely used as the first step in mechanism design that most designers do not stop to question why. Are the precision conditions of greater importance than the other requirements listed? Is a mechanism with branching problems more acceptable than a mechanism that does not satisfy the prescribed motion? The answer to both of these questions, of course, is "no," since a mechanism must satisfy all of the design requirements if it is to be a workable solution. Why then should mechanism design always begin with precision position synthesis? Why not begin designing a mechanism by first considering, for instance, the Grashof condition? From the set of all possible mechanisms of a given type, the subset consisting of only those of the desired Grashof type could be selected. From this subset, mechanisms meeting the specified precision conditions could then be selected, yielding an even smaller subset. Following this, the link length ratio requirements could be applied as a third requirement, and so forth. This approach seems practical enough, and yet, to the author's knowledge, it has never been applied.

The reason for using precision position synthesis as the first step in mechanism design is not that it is the most important consideration. Rather, it is used as a first step because, among all the requirements

listed at the beginning of this section, only the precision position specifications are generally in the form of equality constraints. Thus, it is often possible to use the elimination of variables technique discussed in section 2.4.2. In addition, it may be possible for the designer to choose the number of equality constraints by choosing the number of specified precision positions. This gives the designer control over the number of design variables he will have to work with. Although used intuitively for years, the concept of precision position specifications to reduce the number of free-choice variables was formally advanced by Tesar (26), Eschenbach and Tesar (27), Tesar and Spitznagel (51,52) and Sutherland (63), among others. Of these works, only Sutherland's addresses the possibility of approximately satisfying additional motion requirements.

Two principal drawbacks may exist to using precision condition equality constraints to eliminate variables. First, as discussed in section 2.4.2, this technique introduces nonlinearities which tend to distort the objective function and the remaining constraints. Thus, for example, using a three precision-position solution in which the unknown variables are nonlinear and must be solved for numerically, even though one more variable is eliminated in the latter case. The second drawback is

that the designer may not have a need to satisfy any precision conditions at all, but rather, may require some approximate motion specification to be met. In this case, a great many possible solutions are lost by arbitrarily setting up precision position requirements. If no acceptable design can be found from this reduced solution set, the designer may be forced to eliminate some, or all, of the precision conditions.

Although not so obvious or as well developed in the literature, it may sometimes be possible to use other design requirements as equality constraints to eliminate variables. For example, one variable could be eliminated by specifying the coupler link to be twice as long as the input crank. Or perhaps the minimum and maximum transmission angles could be specified to have a certain numerical value. Again, the designer must be careful not to introduce extreme nonlinearities in the remaining design equations. Although not explored in detail here, this appears to be a novel and potentially useful area of research.

Eschenbach and Tesar (27) have suggested dividing the design requirements into two groups: necessary requirements and desirable requirements. In their work, the necessary requirements are the branching, Grashof and order conditions, plus the requirement of satisfying four precision conditions. The remaining requirements

are treated as desirable conditions. The necessary requirements are considered to be go - no go conditions, because they are either acceptable or they are unacceptable. In other words, these are the constraint conditions. The desirable conditions collectively correspond to the objective function, the idea being to find the most desirable value for a given combination of these conditions.

It is doubtful that the division of necessary and desirable conditions suggested by Eschenbach and Tesar (27) will apply to every mechanism design problem. For example, satisfying four precision positions may well be a desirable condition, while fixed-pivot location may be a necessary condition. However, their work is indicative of the importance placed on satisfying the first four conditions listed at the beginning of this section, namely, motion specification, branch avoidance, order of positions and Grashof's condition.

3.3 Observations and Trends Affecting Mechanism Optimization

The development of a general philosophy for mechanism optimization undoubtedly must involve some amount of subjective opinion and speculation. Some of the concepts presented in this section break away from established trends in mechanism optimization. This is partially due to a reevaluation of mechanism optimization procedures based on past literature, and partially due to the rapidly

changing role of the computer in engineering design. Although it is difficult to make broad generalizations about any subject, the following observations seem to apply:

(1) The need to find a globally optimum solution has, at times, been overplayed in the literature on mechanism optimization. Most, if not all, practical mechanism design problems are of such a complex and multi-faceted nature that it is impossible to precisely define what is meant by optimum. Furthermore, finding the global optimum is not really necessary for many practical mechanism design problems. Therefore, the objective of this dissertation is to use optimization theory to find designs which are workable solutions, rather than emphasizing finding <u>the</u> optimum solution. For this reason, care has been used to call the present work, "Optimization of Spatial Mechanisms," rather than, for example, "Optimum Design of Spatial Mechanisms."

(2) Centralized large-scale digital computers are being replaced in many applications by smaller and more local ones. While the

capabilities of these smaller machines have increased enormously in recent years, storage capacity often limits their usefulness to smaller-scale problems. The trend, therefore, should be toward more compact iteratively-based programs which require less storage.

(3) Since designers will more often be using local, and perhaps personal, computers, the algorithms developed should be easy to program and readily adaptable to a variety of problems. This means that the design theories should, so far as possible, be based on more easily understood concepts.

(4) Based on points (2) and (3) above, the optimization method employed should be simple in concept, require little computer storage, and should be capable of solving a wide variety of problems. The Hooke and Jeeves' method described in Chapter Two possesses these qualities.

3.3 Development of a General Mechanism Optimization Philosophy

In the past, many authors have developed optimization methods which do not include a parameter reduction step,

for example (25,54). A simple flow chart for this method is shown in figure 3.1. These methods might be called design by analysis, because there is no synthesis step involved. The advantage of this approach is that no potential solutions are lost through parameter reduction. Given enough time and computer resources, this would be the preferred method. However, this approach will generally be too inefficient for use on complex mechanism design problems where a large number of variables are present, such as spatial mechanism design.

Undoubtedly, the most popular approach to mechanism optimization has been the "standard" precision position approach shown in figure 3.2. This method is generally more efficient than the design by analysis technique, because the number of free-choice parameters has been reduced. However, this method is still lacking in the respect that no attempt has been made to distinguish nonparametric constraints from parametric ones. In addition, this method does not recognize the possibility of eliminating variables other than by precision position synthesis. Also, no provision is generally included to allow additional approximate motion specifications.

A flowchart depicting the method of sequential filters, as presented by Spitznagel and Tesar (51,52), is shown in figure 3.3. It is an improvement over the

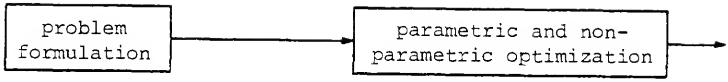


figure 3.1 Design by analysis

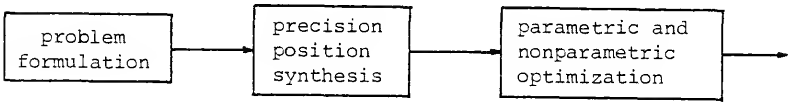


figure 3.2 Standard approach to mechanism optimization

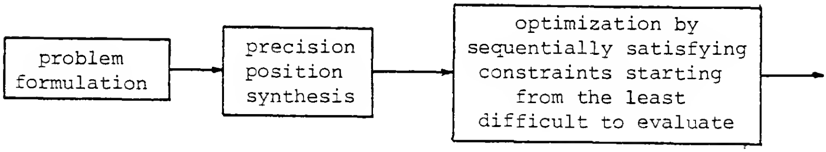


figure 3.3 The method of sequential filters

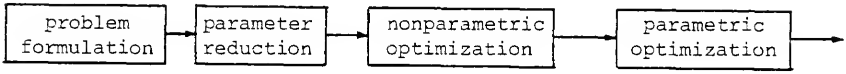


figure 3.4 The mechanism optimization method
used in this dissertation

NOTE: Feedback may occur to any previous block in all figures.

standard approach because it recognizes many of the special properties which result when dealing with a planar four-bar linkage. However, no distinction is made between parametric and nonparametric constraints. Also, only precision position synthesis is considered as a parameter reduction tool and no provision is made for satisfying additional approximate motion specifications.

Sutherland (63) was the first to recognize the potential need for and the usefulness of satisfying both precision and approximate motion specifications simultaneously. Although not a complete optimization scheme in itself, this feature is both a desirable and a necessary component of any general optimization scheme.

The most general and efficient mechanism optimization scheme should utilize the best features from all of the above methods. It should incorporate parameter reduction, either using precision position synthesis or perhaps using other equality constraints. It should also incorporate approximate motion specification in addition to precision conditions. Finally, it should make a distinction between parametric constraints and nonparametric ones, and should use special properties of the mechanism to eliminate parametric constraints, when possible. One such method is illustrated by the flowchart, figure 3.4. While simple (and perhaps obvious) in concept, application of the above-described method

will often require a deep understanding of the specific problem at hand. Spatial mechanism design, the topic of the next four chapters, demonstrates this point. Chapter Four discusses parameter reduction by means of closed-form precision position solutions for various spatial dyads. Chapters Five and Six demonstrate methods for formulating some of the constraints for the RCCC and the RSSR mechanisms in nonparametric form. These are then used, along with the precision position synthesis methods of Chapter Four, to demonstrate the optimization of these mechanisms.

CHAPTER 4

PRECISION POSITION SYNTHESIS OF SPATIAL MECHANISMS

4.1 Introduction to Precision Position Synthesis

The preceding chapters emphasized the need for parameter reduction in mechanism optimization. Clearly, precision position synthesis is often of importance, and offers a readily available means for reducing the number of parameters.

In this chapter, closed-form solutions are developed for rigid-body guidance problems using the RS, CS, CC and RC dyads. Since the function generation problem can generally be converted to a rigid-body guidance problem by a process known as inversion (19), and since the path generation problem can generally be treated as an incompletely specified rigid-body guidance problem, the solutions developed in this chapter have a broad spectrum of applications. Furthermore, as shown in the following section, the dyads treated here can be assembled into a variety of mechanisms, giving the procedures an even wider applicability.

Synthesis procedures for some of the dyads treated in this chapter have been discussed elsewhere (76,102,

110, 111, 112). However, these works generally do not emphasize finding closed-form linear solutions, a topic of considerable importance when the synthesis process is to be used in an optimization loop. A detailed discussion of these synthesis procedures was also felt to be needed in order to provide a uniform notation for later reference and to provide a better understanding of the available free-choice parameters. All of the synthesis procedures given here are for the maximum number of precision positions which result in closed-form linear solutions.

4.2 Dyadic Synthesis of Mechanisms

For the purposes of this work, dyads may be thought of as building blocks for mechanism synthesis. A dyad is a two-link kinematic chain composed of a grounded link and a floating link, and having two degrees of freedom. The grounded link is joined to ground through one kinematic pair and joined to the floating link through another. For simplicity and clarity, the concept of dyadic mechanism synthesis will be explained using the planar dyad shown in figure 4.1. Extension of these concepts to spatial dyads will readily follow.

Begin by considering a set of discrete planar precision positions, defined by $f(x_j, y_j)$, j=1,2,3, as shown in figure 4.1. Any dyad whose tracer point can physically reach all of these positions can generate

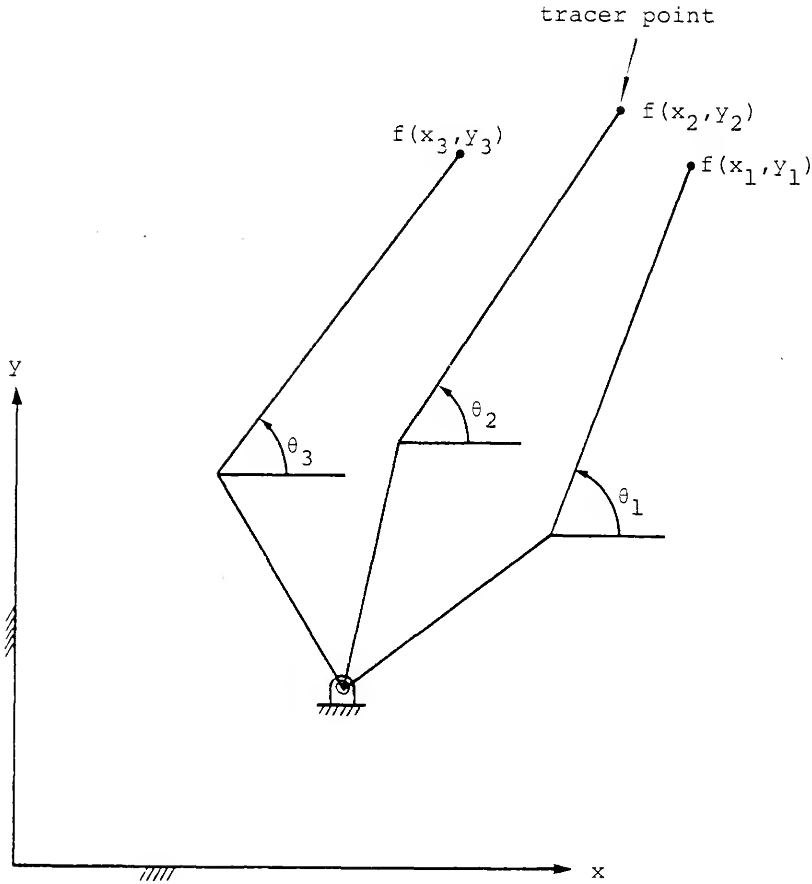


figure 4.1 A planar dyad

these points. In fact, it is possible to generate an infinite number of such positions. If, however, rotations of the floating link or of the grounded link are specified as θ_j, j=1,2,3 at each position, the problem becomes much more difficult -- so much so, in fact, that it is now possible to generate a maximum of only five such positions.

When the rotations of the floating link are specified, the problem becomes identical to the planar rigid-body guidance problem. Classical Burmester theory is a well-known method for solving this problem. If two different dyads can be found which satisfy the given motion requirements, their floating links may be rigidly connected to form a constrained planar four-bar linkage, as shown in figure 4.2.

It should be apparent that spatial mechanisms can be constructed from spatial dyads in a similar fashion. Of course, the problems become more complex since the motions are now in three dimensions, and because a greater number of joint types, or pairs, must be considered. For a discussion of the various types of joints used in spatial mechanisms see Harrisberger (17).

When designing single-input mechanisms, dyads usually must be connected in combinations which result in a single degree of freedom, although a number of exceptions to this rule exist (see, for example, Shigley and Uicker (113)). In any case, several single-input

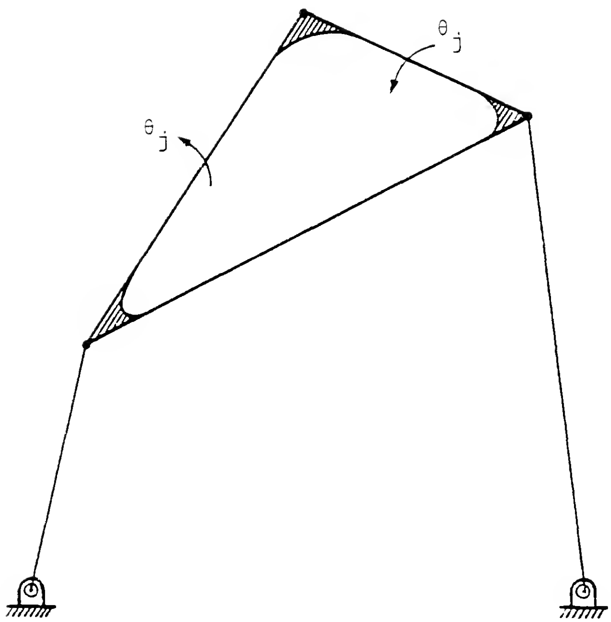


figure 4.2 Two dyads with identical floating link rotations joined to produce a four-bar linkage

spatial mechanisms which can be synthesized from the dyads treated in this chapter are shown in figure 4.3. Thus, for example, if the RC and CC dyads can be independently synthesized for three rigid-body positions, they can be joined together to form an RCCC mechanism also capable of generating these three positions.

4.3 Position and Orientation of a Body in Space

One of the fundamental tools necessary for designing spatial mechanisms is the ability to describe the motion of rigid bodies in space. Since the examples presented in this work deal only with finitely separated positions, the present discussion is concerned with describing finite displacements; extension of this concept to include higher order properties can be found in many standard texts, for example (76).

The total displacement of a rigid body in space can always be considered to be the sum of an angular rotation and a linear displacement of a reference point fixed in the moving body. The displacement of a point in space is easily described by a single three-component vector. However, describing angular displacements in space is not so easy nor is the method so obvious. Some of the most popular methods are (1) Euler angles; (2) angular rotation about an axis in space; (3) prescribed order of rotations about a right-hand set of Cartesian axes;

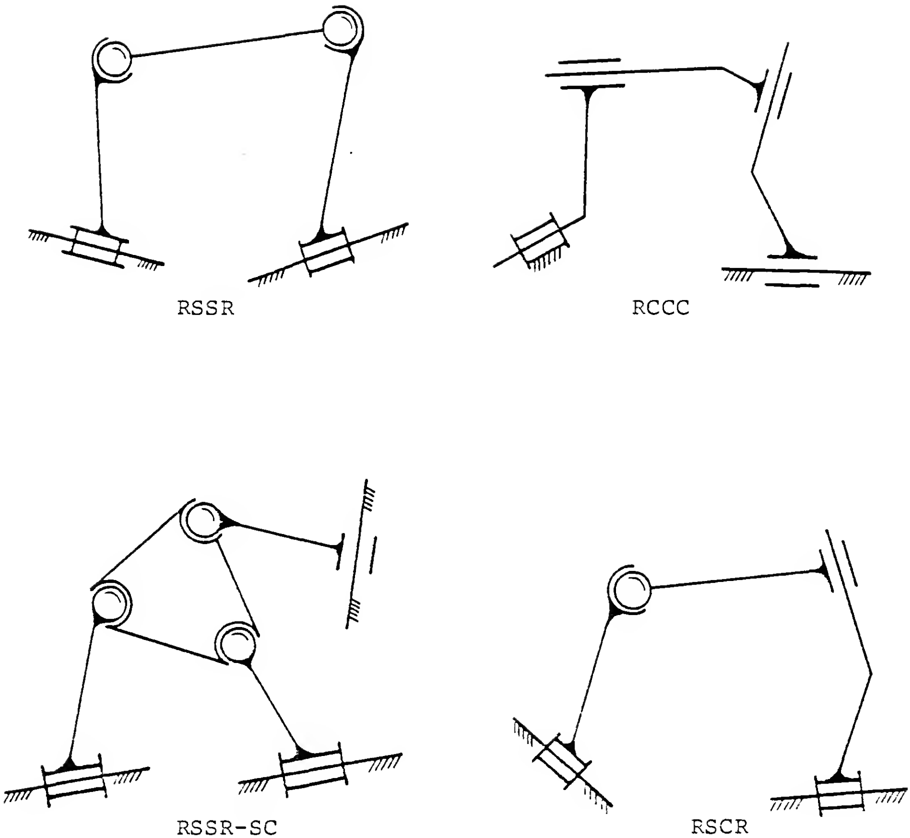


figure 4.3 Some of the dyad-based spatial mechanisms which can be synthesized using the methods of this chapter

and (4) the direction cosines of a pair of independent unit vectors fixed in the body. Simultaneous translation and rotation can be specified in a single quantity in several ways, such as, for example, (1) quaternions (96); (2) 3x3 matrices and tensors with dual numbers (98); (3) 4x4 matrices using homogeneous coordinates (101); and (4) dual quaternions (97), etc. Coaxial translations and rotations (so-called screw displacements) can also be expressed by way of any of these methods.

Regardless of the specific method being used, it is important to realize that only three independent parameters are needed to describe spatial angular displacements. Nevertheless, it is almost always most convenient to work with a nine-component three-by-three matrix when describing spatial angular displacements; this is the so-called rotation matrix. For a given angular displacement of a body, all of the methods listed above will lead to rotation matrices. For this reason, the rotation matrix will be the basic tool used in this dissertation to describe spatial angular displacements. For detailed discussions on finding the rotation matrix from the angular displacement description methods listed above, see (76,114).

The rotation matrix is extremely convenient for specifying finite rotations in space. For example, if $\underset{\sim}{v}_1$ and $\underset{\sim}{v}_2$ represent two arbitrarily oriented spatial

positions of the vector $\underset{\sim}{v}$, then the rotation from position 1 to position 2 can be expressed in the form

$$\underset{\sim}{v}_2 = [R]\, \underset{\sim}{v}_1 \tag{4.1}$$

where [R] is a rotation matrix (76). Equation 4.1 can be expanded in terms of components to give

$$\begin{Bmatrix} v_{2x} \\ v_{2y} \\ v_{2z} \end{Bmatrix} = \begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{bmatrix} \begin{Bmatrix} v_{1x} \\ v_{1y} \\ v_{1z} \end{Bmatrix} \tag{4.2}$$

4.4 Synthesis of the Revolute-Spheric (RS) Dyad

The revolute-spheric dyad (RS dyad for short) is shown schematically in the jth position with associated vectors in figure 4.4. It is one of the simplest dyads to synthesize and is perhaps the most useful. The RS dyad can be synthesized for up to three precision positions of rigid-body guidance using linear, closed-form solution procedures (103).

The following vectors are used in figure 4.4 to describe the RS dyad:

$\underset{\sim}{a}_0$ locates the revolute joint center relative to the fixed coordinate system

$\underset{\sim}{A}_j$ locates the spheric joint center in the jth

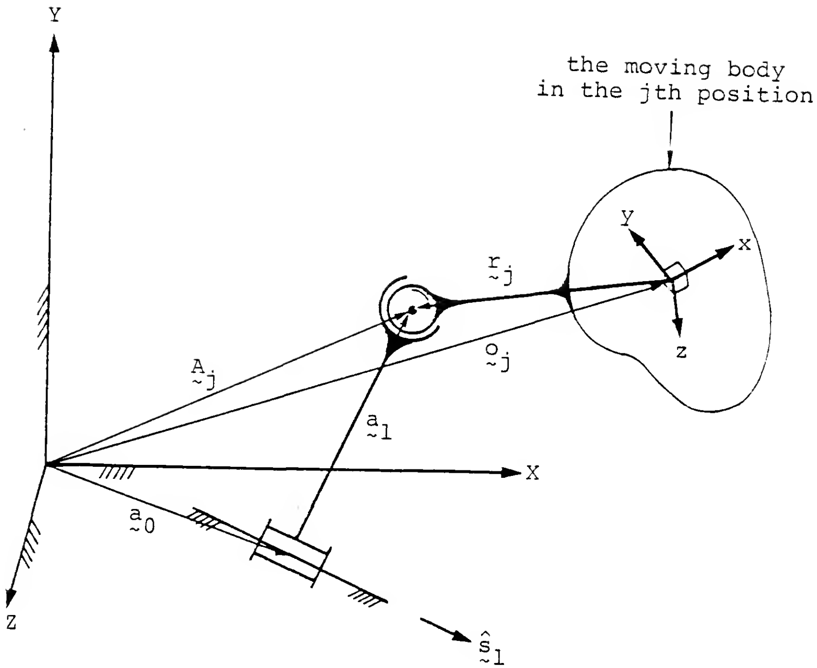


figure 4.4 The RS dyad and associated vectors in the jth position

position relative to the fixed coordinate system

$\underset{\sim}{o}_j$ locates the center of the moving coordinate system relative to the fixed coordinate system

$\underset{\sim}{r}_j$ locates the spheric joint center in the jth position relative to the moving coordinate system

$\hat{\underset{\sim}{s}}_1$ unit vector (denoted by the hat) along the fixed revolute axis

The synthesis procedure can be viewed intuitively as follows. Select an arbitrary point fixed in the moving rigid body as the location of the spheric joint. The location of this point at the three prescribed positions defines a plane (three points define a plane), and a circle lying in this plane (three points also define a circle). Since the revolute joint physically constrains points to lie on a circle, its axis must pass through the center of this circle, and must also be along the normal to the plane containing the three points. The mathematical formulation of this easily visualized procedure is developed below.

The positions of a body in space are specified by giving the location and orientation of the moving coordinate system embedded in the body at each position, as described in section 4.3. Thus, for three prescribed

positions, the given quantities are

$$\underset{\sim}{o}_j \text{ and } [R_j] \qquad j = 1,2,3 \tag{4.3}$$

where $\underset{\sim}{o}_j$ is a three component vector, and $[R_j]$ is the rotation matrix rotating the moving body from a reference position to position j. Displaced, rather than absolute, positions of the moving body are of importance here. Therefore, the starting position, defined by $\underset{\sim}{o}_1$ and $[R_1]$, may be selected arbitrarily, and the other two positions measured relative to the first. Choosing $[R_1]$ to be a 3x3 identity matrix has the effect of making the fixed and moving coordinate systems parallel in the initial position. Since this assumption generally simplifies the design equation, it will be employed throughout the remainder of this chapter. The initial position vector, $\underset{\sim}{o}_1$, may be taken to be any convenient point in the moving body.

Referring to figure 4.4, begin the RS dyad synthesis by assuming a value of $\underset{\sim}{r}_1$, the vector locating the spheric joint relative to the origin of the moving coordinate system in the initial position. Since the locations of the moving origin, $\underset{\sim}{o}_j$, and rotation matrices, $[R_j]$, j = 1,2,3 have been specified, the vector $\underset{\sim}{A}_j$, locating the spheric joint center, can be expressed as

$$\underset{\sim}{A}_j = \underset{\sim}{o}_j + \underset{\sim}{r}_j, \qquad j = 1,2,3 \tag{4.4a}$$

or

$$\underset{\sim}{A}_j = \underset{\sim}{o}_j + [R_j]\underset{\sim}{r}_1, \qquad j = 1,2,3 \tag{4.4b}$$

It has already been noted that the three positions of the spheric joint define a plane whose normal is parallel to the fixed revolute axis $\hat{\underset{\sim}{s}}_1$. Since the vectors $\underset{\sim}{A}_2-\underset{\sim}{A}_1$ and $\underset{\sim}{A}_3-\underset{\sim}{A}_2$ lie in this plane, the unit vector $\hat{\underset{\sim}{s}}_1$ along the revolute axis is defined by

$$\hat{\underset{\sim}{s}}_1 = \frac{(\underset{\sim}{A}_2 - \underset{\sim}{A}_1) \times (\underset{\sim}{A}_3 - \underset{\sim}{A}_2)}{|(\underset{\sim}{A}_2 - \underset{\sim}{A}_1) \times (\underset{\sim}{A}_3 - \underset{\sim}{A}_2)|} \tag{4.5}$$

The plane containing the vectors $\underset{\sim}{A}_2-\underset{\sim}{A}_1$ and $\underset{\sim}{A}_3-\underset{\sim}{A}_2$ and normal to $\hat{\underset{\sim}{s}}_1$ is shown in figure 4.5. It is necessary to determine the vector $\underset{\sim}{a}_0$ locating the intersection of this plane with the fixed revolute axis. This is accomplished by first determining the unit vectors $\hat{\underset{\sim}{p}}_2$ and $\hat{\underset{\sim}{p}}_3$ which are respectively perpendicular to the vectors $\underset{\sim}{A}_2-\underset{\sim}{A}_1$ and $\underset{\sim}{A}_3-\underset{\sim}{A}_2$

$$\hat{\underset{\sim}{p}}_2 = \hat{\underset{\sim}{s}}_1 \times (\underset{\sim}{A}_2 - \underset{\sim}{A}_1)/|\underset{\sim}{A}_2 - \underset{\sim}{A}_1| \tag{4.6}$$

$$\hat{\underset{\sim}{p}}_3 = \hat{\underset{\sim}{s}}_1 \times (\underset{\sim}{A}_3 - \underset{\sim}{A}_2)/|\underset{\sim}{A}_3 - \underset{\sim}{A}_2| \tag{4.7}$$

The perpendicular bisectors of the vectors $\underset{\sim}{A}_2-\underset{\sim}{A}_1$ and $\underset{\sim}{A}_3-\underset{\sim}{A}_2$ intersect at the tip of position vector $\underset{\sim}{a}_0$ (as shown in figure 4.5). Denoting the perpendicular

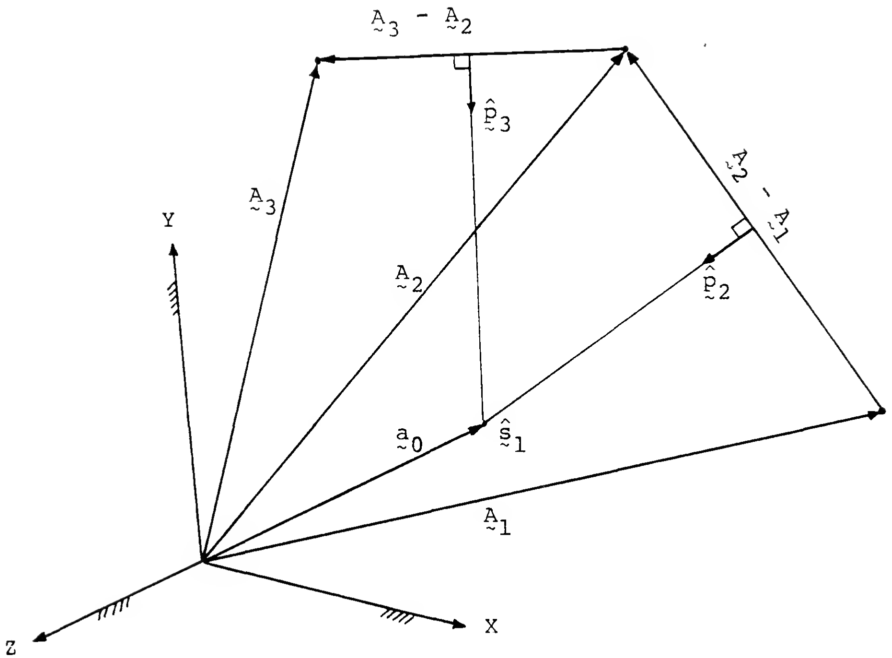


figure 4.5 The plane of the RS dyad

distances from $\mathbf{A}_2-\mathbf{A}_1$ and $\mathbf{A}_3-\mathbf{A}_2$ to the tip of $\mathbf{a}_0$ by λ_2 and λ_3, the vector $\mathbf{a}_0$ can be expressed in the alternate forms

$$\mathbf{a}_0 = \lambda_2\hat{\mathbf{p}}_2 + \mathbf{A}_1 + \frac{\mathbf{A}_2 - \mathbf{A}_1}{2} \tag{4.8}$$

$$\mathbf{a}_0 = \lambda_3\hat{\mathbf{p}}_3 + \mathbf{A}_2 + \frac{\mathbf{A}_3 - \mathbf{A}_2}{2} \tag{4.9}$$

Equating the right sides of equations 4.8 and 4.9 and forming the vector product with $\hat{p}_2$ eliminates λ_2 and yields

$$\cancelto{0}{\lambda_2\hat{\mathbf{p}}_2 \times \hat{\mathbf{p}}_2} + \hat{\mathbf{p}}_2 \times \mathbf{A}_1 + \hat{\mathbf{p}}_2 \times \frac{\mathbf{A}_2 - \mathbf{A}_1}{2} =$$

$$\lambda_3\hat{\mathbf{p}}_2 \times \hat{\mathbf{p}}_3 + \hat{\mathbf{p}}_2 \times \mathbf{A}_2 + \hat{\mathbf{p}}_2 \times \frac{\mathbf{A}_3 - \mathbf{A}_2}{2} \tag{4.10}$$

Equation 4.10 can now be solved explicitly for λ_3 by forming the scalar product with $(\hat{\mathbf{p}}_2 \times \hat{\mathbf{p}}_3)$ which gives

$$\lambda_3 = \frac{(\hat{\mathbf{p}}_2 \times \hat{\mathbf{p}}_3) \cdot (\mathbf{C}_1 + \mathbf{C}_2 - \mathbf{C}_3 - \mathbf{C}_4)}{(\hat{\mathbf{p}}_2 \times \hat{\mathbf{p}}_3) \cdot (\hat{\mathbf{p}}_2 \times \hat{\mathbf{p}}_3)} \tag{4.11}$$

where

$$\mathbf{C}_1 = \hat{\mathbf{p}}_2 \times \mathbf{A}_1 \tag{4.12}$$

$$\mathbf{C}_2 = \hat{\mathbf{p}}_2 \times \frac{\mathbf{A}_2 - \mathbf{A}_1}{2} \tag{4.13}$$

$$\underset{\sim}{C}_3 = \hat{\underset{\sim}{p}}_2 \times \underset{\sim}{A}_2 \tag{4.14}$$

$$\underset{\sim}{C}_4 = \hat{\underset{\sim}{p}}_2 \times \frac{\underset{\sim}{A}_3 - \underset{\sim}{A}_2}{2} \tag{4.15}$$

The value of λ_3 thus obtained can be back substituted into equation 4.9 to determine $\underset{\sim}{a}_0$. The starting position of the grounded link, represented by the vector $\underset{\sim}{a}_1$, can be obtained from the expression

$$\underset{\sim}{a}_1 = \underset{\sim}{A}_1 - \underset{\sim}{a}_0 \tag{4.16}$$

This completely determines the dimensions and the starting position of the RS dyad.

4.5 Synthesis of the Cylindric-Spheric (CS) Dyad

The procedure for synthesizing the CS dyad is similar to the RS dyad synthesis procedure. Additionally, the motion of the grounded cylindric joint along its axis must be considered. This scalar variable is denoted by S_j in figure 4.6, which shows the 1st and jth position of the CS dyad.

The CS dyad synthesis procedure can be visualized as follows. Referring to figure 4.6, once again assume the vector $\underset{\sim}{r}_1$ locating the spheric joint relative to the origin of the moving xyz coordinate system in its initial position. Also, assume the orientation of the

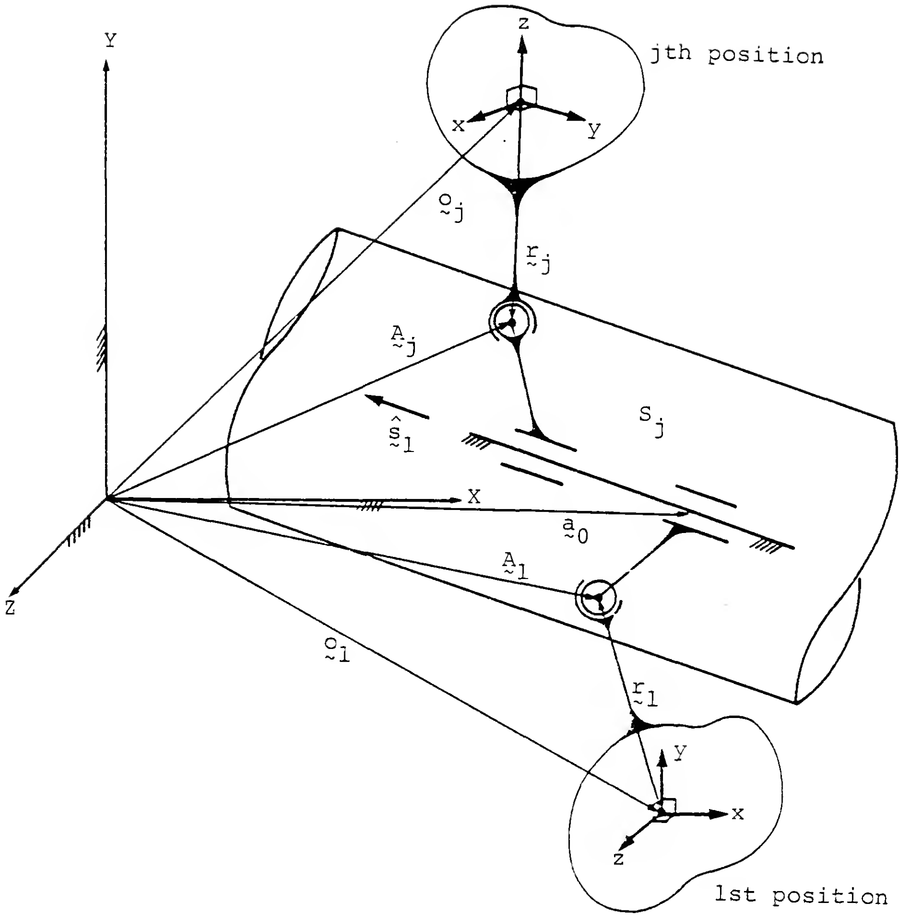


figure 4.6 CS dyad in the 1st and the jth position

cylindric joint axis, $\hat{\underset{\sim}{s}}_1$, in the fixed XYZ coordinate system. Since the spheric joint is physically constrained to lie on a cylinder about $\hat{\underset{\sim}{s}}_1$, projections of its location onto a plane normal to $\hat{\underset{\sim}{s}}_1$ must lie on a circle. The procedure is thus to project the three locations of the spheric joint onto a plane normal to $\hat{\underset{\sim}{s}}_1$. These three points define a circle within the plane, and the remainder of the procedure becomes the same as for the RS dyad. This procedure is described mathematically below.

As before, three positions of the moving body are specified

$$\underset{\sim}{o}_j,\ [R_j], \qquad j = 1,2,3 \tag{4.17}$$

Assuming the vector $\underset{\sim}{r}_1$ arbitrarily again leads to the result

$$\underset{\sim}{A}_j = \underset{\sim}{o}_j + [R_j]\underset{\sim}{r}_1, \qquad j = 1,2,3 \tag{4.18}$$

The orientation of the cylindric joint axis is now assumed arbitrarily by specifying two of its components. The third component can be found from the unit vector identity

$$s_{1x}^2 + s_{1y}^2 + s_{1z}^2 = 1 \tag{4.19}$$

Figure 4.7 shows the vector $\hat{\underset{\sim}{s}}_1$ in true length. The relationship between the vector $\hat{\underset{\sim}{s}}_1$ and the scalar

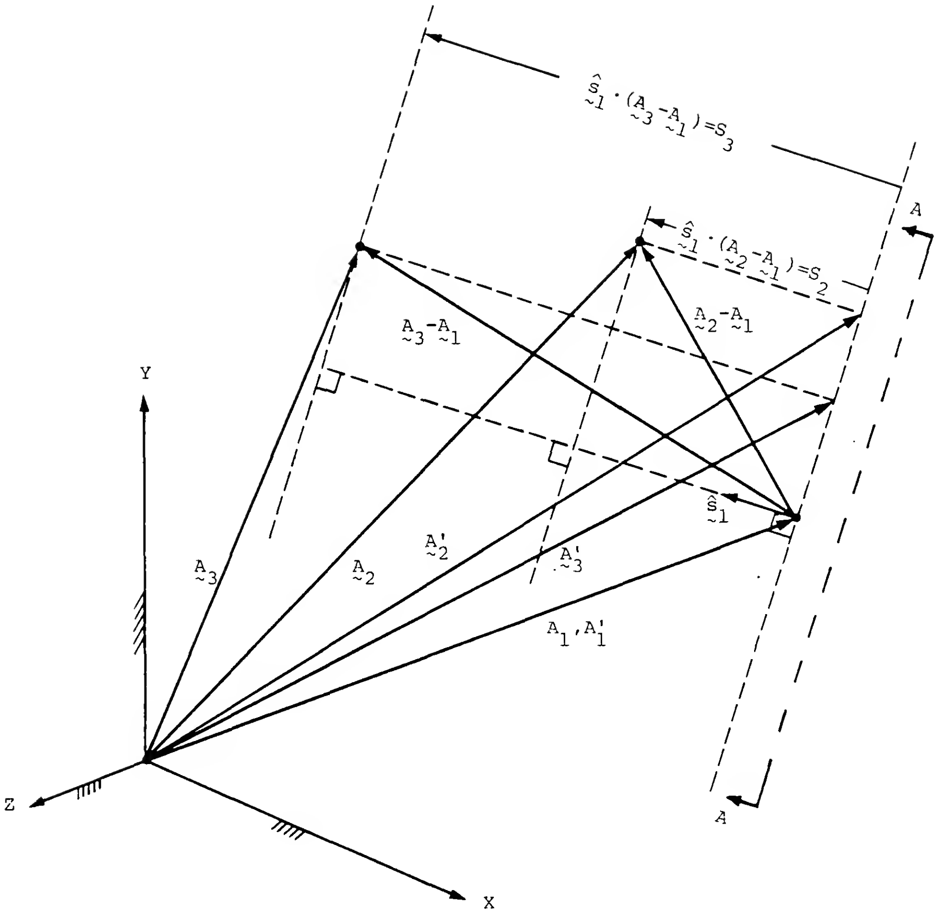


figure 4.7 The CS dyad showing vector $\hat{s}_1$ in true length

displacements S_2 and S_3 can be determined from this figure to be

$$S_2 = \hat{\underset{\sim}{s}}_1 \cdot (\underset{\sim}{A}_2 - \underset{\sim}{A}_1) \tag{4.20}$$

$$S_3 = \hat{\underset{\sim}{s}}_1 \cdot (\underset{\sim}{A}_3 - \underset{\sim}{A}_1) \tag{4.21}$$

At this point it is worthwhile to point out that the arbitrarily prespecified variables, $\underset{\sim}{r}_1$ and $\hat{\underset{\sim}{s}}_1$, are by no means the only possible choices. In fact, for three precision position synthesis, any five scalar variables may be selected arbitrarily. However, the variables selected here are believed to be the most convenient to use in solving the kinematic synthesis problem.

Some argument could be made for preselecting S_2 and S_3 rather than $\hat{\underset{\sim}{s}}_1$ (recall that only two independent scalar quantities comprise $\hat{\underset{\sim}{s}}_1$), and solving equations 4.19 through 4.21 for the components of $\hat{\underset{\sim}{s}}_1$. This is beneficial because the designer has a "feeling" for what values to assign to S_2 and S_3 (constraints would probably be placed on their minimum and maximum values), whereas selection of $\hat{\underset{\sim}{s}}_1$ would be arbitrary. The benefit of having feasible starting values for mechanism optimization, however, might be outweighed by the additional computation required to find $\hat{\underset{\sim}{s}}_1$ at each iteration.

The author's opinion is that these effects would roughly balance, although no test of this has been made.

Figure 4.8 shows section A-A taken from figure 4.7. This is the plane defined by the unit normal $\hat{\underset{\sim}{s}}_1$, and passing through the point A_1. The points A_2' and A_3' are the projections of points A_2 and A_3 onto this plane. The vectors locating these points are given as follows:

$$\underset{\sim}{A}_2' = \underset{\sim}{A}_2 - S_2\hat{\underset{\sim}{s}}_1 \tag{4.22}$$

$$\underset{\sim}{A}_3' = \underset{\sim}{A}_3 - S_3\hat{\underset{\sim}{s}}_1 \tag{4.23}$$

It should be noted that the right-hand sides of these equations are now known, and that $\underset{\sim}{A}_1' = \underset{\sim}{A}_1$.

In the CS dyad, the cylindric joint constrains the spheric joint to move on the surface of a cylinder whose axis is defined by $\hat{\underset{\sim}{s}}_1$. The projections of points on the cylinder onto a plane normal to $\hat{\underset{\sim}{s}}_1$ will, therefore, lie on a circle. Thus, points $\underset{\sim}{A}_1'$ $\underset{\sim}{A}_2'$ and $\underset{\sim}{A}_3'$ define a circle with its center on the line defined by $\hat{\underset{\sim}{s}}_1$.

The problem of determining the initial location of the cylindric joint, $\underset{\sim}{a}_0$, is thus identical to the RS dyad synthesis. Equations 4.6-4.16 apply directly to the CS dyad synthesis, except that $\underset{\sim}{A}_1$, $\underset{\sim}{A}_2$ and $\underset{\sim}{A}_3$ should be replaced everywhere by $\underset{\sim}{A}_1'$, $\underset{\sim}{A}_2'$ and $\underset{\sim}{A}_3'$. The dimensions of the CS dyad are thus completely determined.

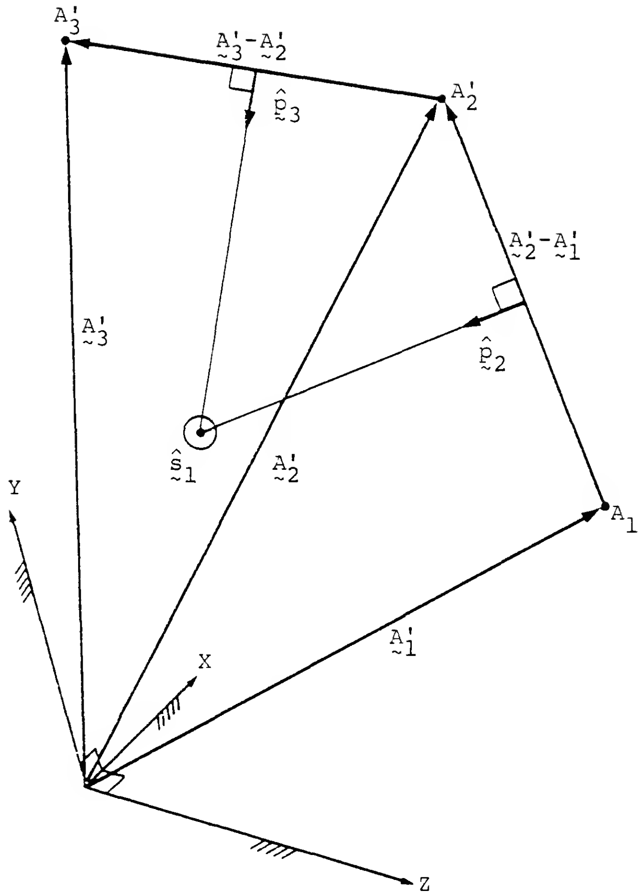


figure 4.8 Section A-A from figure 4.7

4.6 Synthesis of the Cylindric-Cylindric (CC) Dyad

A schematic representation of the CC dyad in the jth position is shown in figure 4.9. The following vector and scalar parameters are used in figure 4.9 to describe the CC dyad. All vectors are expressed relative to the fixed coordinate system unless otherwise stated.

- $\underset{\sim}{a}_0$ locates the grounded cylindric joint in its initial position
- $\underset{\sim}{a}_{0j}$ locates the grounded cylindric joint in its jth position
- $\hat{\underset{\sim}{s}}_1$ unit vector along the fixed axis of the grounded cylindric joint
- $S1_j$ scalar displacement of the grounded cylindric joint along its axis
- $\hat{\underset{\sim}{s}}_{2j}$ unit vector along the ungrounded cylindric joint axis in the jth position
- $\hat{\underset{\sim}{a}}_{12}$ unit vector along the common normal from the fixed to the moving joint axis
- $\underset{\sim}{a}'_j$ locates the jth position of the intersection of the ungrounded joint axis and the common normal to the joint axes
- $\underset{\sim}{a}_j$ locates the ungrounded cylindric joint in the jth position

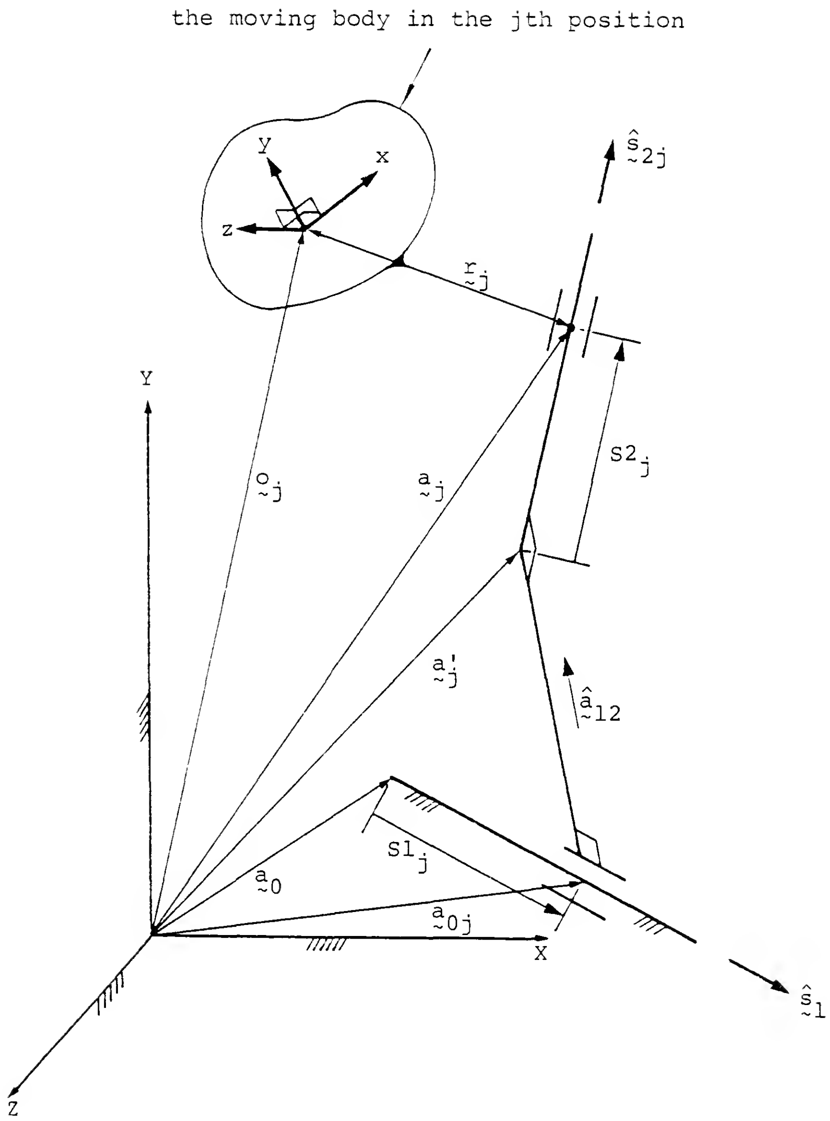


figure 4.9 The CC dyad and associated vectors in the jth position

$S2_j$ scalar displacement of the ungrounded cylindric joint along its axis

$\underset{\sim}{r}_j$ locates the ungrounded cylindric joint relative to the moving (body-fixed) coordinate system

$\underset{\sim}{o}_j$ locates the origin of the moving coordinate system

As before, three prescribed rigid-body positions are given

$$\underset{\sim}{o}_j,\ [R_j],\qquad j = 1,2,3 \tag{4.24}$$

From figure 4.9, the following vector relationships can be deduced:

$$\underset{\sim}{a}_j = \underset{\sim}{o}_j + \underset{\sim}{r}_j \tag{4.25}$$

$$\underset{\sim}{r}_j = [R_j]\underset{\sim}{r}_1 \tag{4.26}$$

$$\hat{\underset{\sim}{s}}_{2j} = [R_j]\hat{\underset{\sim}{s}}_{21} \tag{4.27}$$

$$\underset{\sim}{a}'_j = \underset{\sim}{a}_j - S2_j\hat{\underset{\sim}{s}}_{2j} \tag{4.28}$$

$$\underset{\sim}{a}_{0j} = \underset{\sim}{a}_0 + S1_j\hat{\underset{\sim}{s}}_1 \tag{4.29}$$

It should be noted that the joint displacements $S1_j$ and $S2_j$ may be set equal to zero in the initial position without loss of generality.

It should be pointed out that, unlike the two previous synthesis cases, it is quite difficult to visualize the CC dyad synthesis. For this reason, the approach that will be taken is to solve a set of simultaneous equations derived by considering the physical constraints that exist between the links of the dyad. These constraint equations are enumerated below.

(1) Plane equations: these require the link defined by unit vector $\hat{\underset{\sim}{a}}_{12}$ to be the perpendicular bisector of the joint axes.

$$\hat{\underset{\sim}{s}}_1 \cdot (\underset{\sim}{a}'_j - \underset{\sim}{a}_{0j}) = 0, \qquad j = 1,2,3 \qquad (4.30)$$

$$\hat{\underset{\sim}{s}}_{2j} \cdot (\underset{\sim}{a}'_j - \underset{\sim}{a}_{0j}) = 0, \qquad j = 1,2,3 \qquad (4.31)$$

(2) Constant twist equations: express the requirement that the twist angle between the moving axis, $\hat{\underset{\sim}{s}}_{2j}$, and the fixed axis, $\hat{\underset{\sim}{s}}_1$, remains constant.

$$\hat{\underset{\sim}{s}}_{2j} \cdot \hat{\underset{\sim}{s}}_1 = \hat{\underset{\sim}{s}}_{21} \cdot \hat{\underset{\sim}{s}}_1, \qquad j = 2,3 \qquad (4.32)$$

(3) Constant moment equations: express the requirement that the moment of vector $\hat{\underset{\sim}{s}}_{2j}$ about the $\hat{\underset{\sim}{s}}_1$ axis must be constant.

$$\hat{\underset{\sim}{s}}_1 \cdot (\underset{\sim}{a}_j - \underset{\sim}{a}_{0j}) \times \hat{\underset{\sim}{s}}_{2j} =$$

$$\hat{\underset{\sim}{s}}_1 \cdot (\underset{\sim}{a}_1 - \underset{\sim}{a}_0) \times \hat{\underset{\sim}{s}}_{21}, \qquad j = 2,3 \tag{4.33}$$

Condition (3) above could be replaced by a constant link-length equation of the form

$$(\underset{\sim}{a}_j - \underset{\sim}{a}_{0j}) \cdot (\underset{\sim}{a}_j - \underset{\sim}{a}_{0j}) =$$

$$(\underset{\sim}{a}_1 - \underset{\sim}{a}_{01}) \cdot (\underset{\sim}{a}_1 - \underset{\sim}{a}_{01}), \qquad j = 2,3 \tag{4.34}$$

However, equation 4.33 leads to an easier solution because it is linear in the components of the vectors $\underset{\sim}{a}_j$, $\underset{\sim}{a}_{0j}$, $j = 1,2,3$, whereas equation 4.34 is quadratic in these components.

For three position synthesis of the CC dyad, fourteen unknown parameters exist: $\underset{\sim}{a}_0$, $\underset{\sim}{a}_1$, $\hat{\underset{\sim}{s}}_1$, $\hat{\underset{\sim}{s}}_{21}$, $S1_2$, $S1_3$, $S2_2$, $S2_3$ (recall that $\hat{\underset{\sim}{s}}_1$ and $\hat{\underset{\sim}{s}}_{21}$ are unit vectors containing two independent parameters each). Equations 4.30 through 4.33 represent a total of ten scalar equations. This means that four scalar parameters may be chosen arbitrarily. The solution strategy is as follows:

(1) Assume $\hat{\underset{\sim}{s}}_1$, then solve for $\hat{\underset{\sim}{s}}_{21}$ from equation 4.32, noting that $\hat{\underset{\sim}{s}}_{2j} = [R_j]\hat{\underset{\sim}{s}}_{21}$ and that $\hat{\underset{\sim}{s}}_{21} \cdot \hat{\underset{\sim}{s}}_{21} = 1$.

(2) Expand the remaining eight equations (equations 4.30, 4.31 and 4.33) in terms of the ten unknown quantities ($\underset{\sim}{a}_0$, $\underset{\sim}{a}_1$, $S1_2$, $S1_3$, $S2_2$, $S2_3$) by substituting for other unknown quantities from equations 4.25 through 4.29.

(3) Next, arbitrarily assume $S1_2$ and $S1_3$, and solve the resulting set of eight linear equations in eight unknowns.

The details of this procedure are given below.

Assume the orientation of the fixed cylindric joint axis $\hat{\underset{\sim}{s}}_1 = s_{1x}\hat{\underset{\sim}{i}} + s_{1y}\hat{\underset{\sim}{j}} + s_{1z}\hat{\underset{\sim}{k}}$. Rewriting equations 4.32 with j = 2,3

$$\hat{\underset{\sim}{s}}_{22} \cdot \hat{\underset{\sim}{s}}_1 = \hat{\underset{\sim}{s}}_{21} \cdot \hat{\underset{\sim}{s}}_1 \tag{4.35}$$

$$\hat{\underset{\sim}{s}}_{23} \cdot \hat{\underset{\sim}{s}}_1 = \hat{\underset{\sim}{s}}_{21} \cdot \hat{\underset{\sim}{s}}_1 \tag{4.36}$$

From equation 4.27, $\hat{\underset{\sim}{s}}_{22}$ and $\hat{\underset{\sim}{s}}_{23}$ can be expressed in terms of $\hat{\underset{\sim}{s}}_{21}$

$$\hat{\underset{\sim}{s}}_{22} = [R_2]\hat{\underset{\sim}{s}}_{21} \tag{4.37}$$

$$\hat{\underset{\sim}{s}}_{23} = [R_3]\hat{\underset{\sim}{s}}_{21} \tag{4.38}$$

where $[R_2]$ and $[R_3]$ are 3x3 rotation matrices of the form

$$[R_2] = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \tag{4.39}$$

$$[R_3] = \begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{bmatrix} \tag{4.40}$$

Substituting equation 4.37 into 4.35 and substituting equation 4.38 into 4.36 gives

$$[R_2]\hat{\underset{\sim}{s}}_{21} \cdot \hat{\underset{\sim}{s}}_1 = \hat{\underset{\sim}{s}}_{21} \cdot \hat{\underset{\sim}{s}}_1 \tag{4.41}$$

$$[R_3]\hat{\underset{\sim}{s}}_{21} \cdot \hat{\underset{\sim}{s}}_1 = \hat{\underset{\sim}{s}}_{21} \cdot \hat{\underset{\sim}{s}}_1 \tag{4.42}$$

Expanding these equations into components using $\hat{\underset{\sim}{s}}_{21} = s_{21x}\hat{\underset{\sim}{i}} + s_{21y}\hat{\underset{\sim}{j}} + s_{21z}\hat{\underset{\sim}{k}}$ and $\hat{\underset{\sim}{s}}_1 = s_{1x}\hat{\underset{\sim}{i}} + s_{1y}\hat{\underset{\sim}{j}} + s_{1z}\hat{\underset{\sim}{k}}$ and substituting for $[R_2]$ and $[R_3]$ from equations 4.39 and 4.40 yields, after some manipulation

$$As_{21x} + Bs_{21y} + Cs_{21z} = 0 \tag{4.43}$$

$$Ds_{21x} + Es_{21y} + Fs_{21z} = 0 \tag{4.44}$$

where

$$A = (a_{11} - 1)s_{1x} + a_{21}s_{1y} + a_{31}s_{1z} \tag{4.45}$$

$$B = a_{12}s_{1x} + (a_{22} - 1)s_{1y} + a_{32}s_{1z} \tag{4.46}$$

$$C = a_{13}s_{1x} + a_{23}s_{1y} + (a_{33} - 1)s_{1z} \tag{4.47}$$

$$D = (b_{11} - 1)s_{1x} + b_{21}s_{1y} + b_{31}s_{1z} \tag{4.48}$$

$$E = b_{12}s_{1x} + (b_{22} - 1)s_{1y} + b_{32}s_{1z} \tag{4.49}$$

$$F = b_{13}s_{1x} = b_{23}s_{1y} + (b_{33} - 1)s_{1z} \tag{4.50}$$

Since the a's and b's are prescribed components of the rotation matrices, and since $\underset{\sim}{\hat{s}}_1 = s_{1x}\underset{\sim}{\hat{i}} + s_{1y}\underset{\sim}{\hat{j}} + s_{1z}\underset{\sim}{\hat{k}}$ has been assumed arbitrarily, the values of the scalar constants A through F may be calculated. Therefore, equations 4.43 and 4.44 are linear in the unknown components of $\underset{\sim}{\hat{s}}_{21}$. A third equation is obtained using the expression

$$\underset{\sim}{\hat{s}}_{21} \cdot \underset{\sim}{\hat{s}}_{21} = 1 \tag{4.51}$$

or, in terms of components

$$s_{21x}^2 + s_{21y}^2 + s_{21z}^2 = 1 \tag{4.52}$$

Solving equations 4.43, 4.44 and 4.52 for s_{21z} gives

$$s_{21z} = \pm \sqrt{\frac{1}{\left(\frac{EC - FB}{DB - EA}\right)^2 + \left(\frac{DC - FA}{EA - DB}\right)^2 + 1}} \tag{4.53}$$

and s_{21x} and s_{21y} are given by

$$s_{21x} = \left[\frac{EC - FB}{DB - EA}\right] s_{21z} \tag{4.54}$$

$$s_{21y} = \left[\frac{DC - FA}{EA - DB}\right] s_{21z} \tag{4.55}$$

The direction of the moving cylindric joint axis in its initial position is now completely determined. It should be noted that s_{21z} and hence s_{21x} and s_{21y} will have two values, corresponding to the two roots of equation 4.53. Either one of these two roots can be used because the joint axis is determined by the direction of the vector and its sense is of no importance.

The direction of the moving joint axis, $\hat{\underset{\sim}{s}}_{21}$, has been determined using the assumed direction $\hat{\underset{\sim}{s}}_1$ together with the rotation matrices $[R_2]$ and $[R_3]$. This is important in the design of the RCCC mechanism (see Chapter Five) since Grashof and branching conditions for this mechanism depend only on the directions of the joint axes.

Returning to the CC dyad synthesis, the following vector relations are obtained from equations 4.25 through 4.29.

$$\underset{\sim}{a}_1 = \underset{\sim}{o}_1 + \underset{\sim}{r}_1 \tag{4.56}$$

$$\underset{\sim}{a}_2 = \underset{\sim}{o}_2 + \underset{\sim}{r}_2 = \underset{\sim}{o}_2 + [R_2]\underset{\sim}{r}_1 \tag{4.57}$$

$$\underset{\sim}{a}_3 = \underset{\sim}{o}_3 + \underset{\sim}{r}_3 = \underset{\sim}{o}_3 + [R_3]\underset{\sim}{r}_1 \tag{4.58}$$

$$\underset{\sim}{a}'_1 = \underset{\sim}{a}_1 = \underset{\sim}{o}_1 + \underset{\sim}{r}_1 \tag{4.59}$$

$$\underset{\sim}{a}'_2 = \underset{\sim}{a}_2 - S2_2\hat{\underset{\sim}{s}}_2 = \underset{\sim}{o}_2 + [R_2]\underset{\sim}{r}_1 - S2_2[R_2]\hat{\underset{\sim}{s}}_1 \tag{4.60}$$

$$\underset{\sim}{a}'_3 = \underset{\sim}{a}_3 - S2_3\hat{\underset{\sim}{s}}_2 = \underset{\sim}{o}_3 + [R_3]\underset{\sim}{r}_1 - S2_3[R_3]\hat{\underset{\sim}{s}}_1 \tag{4.61}$$

$$\underset{\sim}{a}_{01} = \underset{\sim}{a}_0 \tag{4.62}$$

$$\underset{\sim}{a}_{02} = \underset{\sim}{a}_0 + S1_2\hat{\underset{\sim}{s}}_1 \tag{4.63}$$

$$\underset{\sim}{a}_{03} = \underset{\sim}{a}_0 + S1_3\hat{\underset{\sim}{s}}_1 \tag{4.64}$$

Substituting these results into the remaining constraint equations (equations 4.30, 4.31 and 4.32) gives

$$\hat{\underset{\sim}{s}}_1 \cdot (\underset{\sim}{o}_1 + \underset{\sim}{r}_1 - \underset{\sim}{a}_0) = 0 \tag{4.65}$$

$$\hat{\underset{\sim}{s}}_1 \cdot (\underset{\sim}{o}_2 + [R_2]\underset{\sim}{r}_1 - S2_2[R_2]\hat{\underset{\sim}{s}}_{21} - \underset{\sim}{a}_0 - S1_2\hat{\underset{\sim}{s}}_1) = 0 \tag{4.66}$$

$$\hat{\underset{\sim}{s}}_1 \cdot (\underset{\sim}{o}_3 + [R_3]\underset{\sim}{r}_1 - S2_3[R_3]\hat{\underset{\sim}{s}}_{21} - \underset{\sim}{a}_0 - S1_3\hat{\underset{\sim}{s}}_1) = 0 \tag{4.67}$$

$$\hat{\underset{\sim}{s}}_{21} \cdot (\underset{\sim}{o}_1 + \underset{\sim}{r}_1 - \underset{\sim}{a}_0) = 0 \tag{4.68}$$

$$([R_2]\hat{\underset{\sim}{s}}_{21}) \cdot (\underset{\sim}{o}_2 + [R_2]\underset{\sim}{r}_1 - S2_2[R_2]\hat{\underset{\sim}{s}}_{21} - \underset{\sim}{a}_0 - S1_2\hat{\underset{\sim}{s}}_1) = 0 \tag{4.69}$$

$$([R_3]\hat{\underset{\sim}{s}}_{21}) \cdot (\underset{\sim}{o}_3 + [R_3]\underset{\sim}{r}_1 - S2_2[R_3]\hat{\underset{\sim}{s}}_{21} - \underset{\sim}{a}_0 - S1_3\hat{\underset{\sim}{s}}_1) = 0 \tag{4.70}$$

$$\underset{\sim}{\hat{s}}_1 \cdot (\underset{\sim}{o}_2 + [R_2]\underset{\sim}{r}_1 - S2_2[R_2]\underset{\sim}{\hat{s}}_{21} - \underset{\sim}{a}_0 - S1_2\underset{\sim}{\hat{s}}_1) \times$$

$$[R_2]\underset{\sim}{\hat{s}}_{21} = \underset{\sim}{\hat{s}}_1 \cdot (\underset{\sim}{o}_1 + \underset{\sim}{r}_1 - \underset{\sim}{a}_0) \times \underset{\sim}{\hat{s}}_{21} \qquad (4.71)$$

$$\underset{\sim}{\hat{s}}_1 \cdot (\underset{\sim}{o}_3 + [R_3]\underset{\sim}{r}_1 - S2_3[R_3]\underset{\sim}{\hat{s}}_{21} - \underset{\sim}{a}_0 - S1_3\underset{\sim}{\hat{s}}_1) \times$$

$$[R_3]\underset{\sim}{\hat{s}}_{21} = \underset{\sim}{\hat{s}}_1 \cdot (\underset{\sim}{o}_1 + \underset{\sim}{r}_1 - \underset{\sim}{a}_0) \times \underset{\sim}{\hat{s}}_{21} \qquad (4.72)$$

If $S1_2$ and $S1_3$ are assumed arbitrarily, equations 4.65 through 4.72 form a set of eight scalar equations in the eight unknown parameters $\underset{\sim}{r}_1$, $\underset{\sim}{a}_0$, $S2_2$ and $S2_3$ (all other parameters are either prescribed or have already been calculated). Expanding these equations using the rules of dot-product and vector cross-product multiplication will show them to be linear in the unknown quantities. The result of this expansion is shown in matrix form in figure 4.10. Although this appears on the surface to be somewhat complex, it should be remembered that the 8x8 coefficient matrix and the 8x1 column vector on the right of the equality sign will simply be arrays of numbers. The solution vector can, therefore, be easily found using any of the standard methods, such as Gaussian elimination or matrix inversion.

Upon solving the above derived set of equations, the dimensions and starting position of the CC dyad are completely determined.

$$
\begin{array}{c}
\begin{array}{cccc} \text{columns 1-3} & \qquad\qquad \text{columns 4-6} & \qquad \text{column 7} & \quad \text{column 8} \end{array} \\
\left[\begin{array}{c:c:c:c}
\hat{\mathbf{s}}_1 & -\hat{\mathbf{s}}_1 & 0 & 0 \\
[R_2]^T\hat{\mathbf{s}}_1 & -\hat{\mathbf{s}}_1 & \hat{\mathbf{s}}_1\cdot([R_2]\hat{\mathbf{s}}_{21}) & 0 \\
[R_3]^T\hat{\mathbf{s}}_1 & -\hat{\mathbf{s}}_1 & 0 & \hat{\mathbf{s}}_1\cdot([R_3]\hat{\mathbf{s}}_{21}) \\
\hat{\mathbf{s}}_{21} & -\hat{\mathbf{s}}_1 & 0 & 0 \\
\hat{\mathbf{s}}_{21} & -([R_2]\hat{\mathbf{s}}_{21}) & -1 & 0 \\
\hat{\mathbf{s}}_{21} & -([R_3]\hat{\mathbf{s}}_{21}) & 0 & -1 \\
[R_2]^T\ ([R_2]\hat{\mathbf{s}}_{21})\times\hat{\mathbf{s}}_1 \quad -\hat{\mathbf{s}}_{21}\times\hat{\mathbf{s}}_1 & -([R_2]\hat{\mathbf{s}}_{21})\times\hat{\mathbf{s}}_1-(\hat{\mathbf{s}}_{21}\times\hat{\mathbf{s}}_1) & 0 & 0 \\
[R_3]^T\ ([R_3]\hat{\mathbf{s}}_{21})\times\hat{\mathbf{s}}_1 \quad -\hat{\mathbf{s}}_{21}\times\hat{\mathbf{s}}_1 & -([R_3]\hat{\mathbf{s}}_{21})\times\hat{\mathbf{s}}_1-(\hat{\mathbf{s}}_{21}\times\hat{\mathbf{s}}_1) & 0 & 0
\end{array}\right]
\end{array}
\begin{array}{c} \\ \left[\begin{array}{c} r_{1x} \\ r_{1y} \\ r_{1z} \\ a_{0x} \\ a_{0y} \\ a_{0z} \\ S2_2 \\ S2_3 \end{array}\right] \end{array}
=
\begin{array}{c} \\ \left[\begin{array}{c}
-\hat{\mathbf{s}}_1\cdot\mathbf{o}_1 \\
S1_2-\hat{\mathbf{s}}_1\cdot\mathbf{o}_2 \\
S1_3-\hat{\mathbf{s}}_1\cdot\mathbf{o}_3 \\
-\hat{\mathbf{s}}_{21}\cdot\mathbf{o}_1 \\
([R_2]\hat{\mathbf{s}}_{21})\cdot S1_2\hat{\mathbf{s}}_1-([R_2]\hat{\mathbf{s}}_{21})\cdot\mathbf{o}_2 \\
([R_3]\hat{\mathbf{s}}_{21})\cdot S1_3\hat{\mathbf{s}}_1-([R_3]\hat{\mathbf{s}}_{21})\cdot\mathbf{o}_3 \\
-\hat{\mathbf{s}}_1\cdot\mathbf{o}_2\times([R_2]\hat{\mathbf{s}}_{21})+\hat{\mathbf{s}}_1\cdot\mathbf{o}_1\times\hat{\mathbf{s}}_{21} \\
-\hat{\mathbf{s}}_1\cdot\mathbf{o}_3\times([R_3]\hat{\mathbf{s}}_{21})+\hat{\mathbf{s}}_1\cdot\mathbf{o}_1\times\hat{\mathbf{s}}_{21}
\end{array}\right] \end{array}
$$

figure 4.10 CC dyad synthesis equations; also valid for the RC dyad by setting $S1_2$ and $S1_3$ equal to zero

4.7 Synthesis of the Revolute-Cylindric (RC) Dyad

Synthesis of the RC dyad follows simply and directly from the foregoing synthesis of the CC dyad. It is apparent that the RC dyad is identical to the CC dyad with one degree of freedom removed, namely, the translation along the grounded cylindric joint axis. Referring to the schematic of the CC dyad shown in figure 4.8, it is seen that the parameter Sl_j will be equal to zero for all possible positions of the RC dyad. Fortunately, in synthesizing the CC dyad, Sl_2 and Sl_3 are free-choice (arbitrarily assumed) parameters. As a result, the RC dyad synthesis equations are the same as the CC dyad synthesis equations, except that Sl_2 and Sl_3 must be set equal to zero. Also recall that Sl_1 is, by definition, equal to zero.

It is important to realize that setting Sl_2 and Sl_3 equal to zero only insures zero displacement along the cylindric joint axis at these two positions. It is, therefore, necessary to physically constrain the revolute joint from moving along its axis.

4.8 Conclusions of Precision Position Synthesis

Methods have been presented for synthesizing RS, CS, CC and RC dyads to meet three prescribed precision positions of spatial rigid-body guidance.

In each case, three is the maximum number of such positions which result in linear closed-form solutions.

An extremely important point to be made here is that the use of these closed-form synthesis techniques does not limit the number of prescribed positions to three. For example, if only two precision positions are specified, the third position can be considered to be a set of six variable parameters in the optimization scheme (notice that six parameters will thus be reintroduced in the design equations). If more than three precision positions are specified, the requirement that these be satisfied can be incorporated in the objective function or the constraint equations of the optimization procedure. Of course, only a limited number of precision conditions can be satisfied. Beyond this number, only approximate solutions can generally be found.

The advantages of the vector-based approach presented here include generality and extendability and, of course, the widespread familiarity engineers have with vector methods. The generality and extendability of this approach can be seen from the work of Hernandez (115), who used vector constraint equations in synthesizing spatial mechanisms containing higher pair joints, such as sphere-plane and cylinder-plane pair joints.

The next two chapters demonstrate the use of the synthesis techniques developed in this chapter as a parameter reduction tool in the optimization process.

CHAPTER 5

OPTIMIZATION OF THE RCCC MECHANISM

5.1 Problem Definition

Although the RCCC mechanism is well known to kinematicians, few examples can be found of its practical application. Potentially, it is useful for the generation of path and rigid-body motions and also for the generation of variable-pitch screws (73).

A great deal of work remains to be done before design theories for the RCCC mechanism are complete to the point where all the design requirements discussed in Chapter Three can be included. The example of this chapter does, however, demonstrate the design of an RCCC mechanism to meet rigid-body motion requirements while being free from branch, order, link-length ratio and fixed pivot location problems and having complete input crank rotatability. It is believed that the solution of this problem represents a significant step forward in the state-of-the-art of design of this mechanism.

5.2 Satisfying Additional Motion Requirements

Sections 4.6 and 4.7 presented linear closed-form methods for synthesizing CC and RC dyads to satisfy three positions of rigid-body guidance. Once synthesized, these dyads may be joined together to form an RCCC mechanism capable of satisfying the three specified positions. Often, however, it is desirable to satisfy more than three positions. Unfortunately, the RC dyad cannot in general be synthesized for more than three precision positions of rigid-body motion generation (76). The CC dyad can be synthesized for a maximum of five exact rigid-body positions. However, the synthesis equations are nonlinear for both four and five positions and, as a result, numerical solution procedures are required in each case (76).

Although more than three exact positions of rigid-body guidance will not generally be obtainable with the RCCC mechanism, a dyad-based procedure will now be described which allows additional positions to be satisfied in an approximate sense. Since the RC dyad synthesis procedures follow directly from the CC dyad synthesis procedures (see Chapter Four), the method described below will be for the CC dyad.

Suppose that four rigid-body precision positions have been specified as

$$\underset{\sim}{o}_j, \ [R_j], \qquad j = 1,2,3,4 \tag{5.1}$$

This four-position motion specification is now broken into two three-position motion specifications

$$\underset{\sim}{o}_j, \ [R_j], \qquad j = 1,2,3 \tag{5.2}$$

$$\underset{\sim}{o}_j, \ [R_j], \qquad j = 1,2,4 \tag{5.3}$$

Now refer to the three-position synthesis procedure for the CC dyad described in section 4.6. The direction of the grounded joint axis, $\hat{\underset{\sim}{s}}_1$, and the scalar displacements $S1_2$ and $S1_3$ are assumed arbitrarily. Substituting these assumed values into the synthesis equations yields the complete dimensions and starting position of the CC dyad. Notice that, for the same values of $\hat{\underset{\sim}{s}}_1$, $S1_2$ and $S1_3$, the motion specifications given by equations 5.2 and 5.3 will generally result in different solution dyads. If, however, a set of values for $\hat{\underset{\sim}{s}}_1$, $S1_2$ and $S1_3$ can be found which yield the same CC dyad from both motion specifications, the resulting dyad will exactly generate all four prescribed positions of equation 5.1. Referring to figure 4.9, it can be seen that the two dyads will be the same provided the vectors $\underset{\sim}{a}_0$, $\hat{\underset{\sim}{s}}_{21}$ and $\underset{\sim}{r}_1$ are the same for each dyad. Using the superscripts 5.2 and 5.3 to denote which motion specification the

above vectors correspond to, the constraint equations for four-position synthesis may be written as

$$\underset{\sim}{a}_0^{5.2} - \underset{\sim}{a}_0^{5.3} = 0 \tag{5.4}$$

$$\hat{\underset{\sim}{s}}_{21}^{5.2} - \hat{\underset{\sim}{s}}_{21}^{5.3} = 0 \tag{5.5}$$

$$\underset{\sim}{r}_1^{5.2} - \underset{\sim}{r}_1^{5.3} = 0 \tag{5.6}$$

If one of the prescribed positions is an approximate rather than a precision motion specification, the above equalities need not strictly apply. Instead, the solution can be cast in the form of a function whose value is to be minimized. One possibility for such a function is

$$F(\hat{\underset{\sim}{s}}_1, Sl_2, Sl_3) = W_1 |\underset{\sim}{a}_0^{5.2} - \underset{\sim}{a}_0^{5.3}|$$

$$+ W_2 |\hat{\underset{\sim}{s}}_{21}^{5.2} - \hat{\underset{\sim}{s}}_{21}^{5.3}| + W_3 |\underset{\sim}{r}_1^{5.2} - \underset{\sim}{r}_1^{5.3}| \tag{5.7}$$

where the absolute value signs denote the magnitude of the vector, i.e., $|\underset{\sim}{a}| = (\underset{\sim}{a} \cdot \underset{\sim}{a})^{\frac{1}{2}}$, and where W_1, W_2 and W_3 are positive weighting factors which determine the relative importance of each term.

Clearly, when F equals zero both CC dyads are the same, and all the prescribed positions are satisfied exactly. When F is greater than zero, the dyads are not the same, and only three of the four positions are

satisfied exactly. If $\underset{\sim}{a}_0^{5.2}$, $\hat{\underset{\sim}{s}}_{21}^{5.2}$ and $\underset{\sim}{r}_1^{5.2}$ are taken as the solution, positions 1,2 and 3 will be exactly satisfied and position 4 will be approximately satisfied. On the other hand, if $\underset{\sim}{a}_0^{5.3}$, $\hat{\underset{\sim}{s}}_{21}^{5.3}$ and $\underset{\sim}{r}_1^{5.3}$ are taken as the solution, positions 1,2 and 4 will be exactly satisfied and position 3 will be approximately satisfied.

The synthesis procedure described above will hereafter be referred to as the method of <u>position stacking</u>. As far as the author is aware, no published work exists on the use of this method for the partially-exact, partially-approximate synthesis of either spatial or planar mechanisms. The method of position stacking has several advantages. First, it retains the closed-form parameter reduction, which greatly reduces the number of components in the design vector. Second, it can easily be extended to include any number of positions. For example, nine positions could be specified in three groups of three. A third advantage is that the maximum number of precision positions can be exactly satisfied without deriving and numerically solving a set of non-linear equations. A fourth advantage of this method is that it can be applied to any dyad for which a closed-form solution is available.

The position stacking method differs from the partially-exact, partially approximate synthesis method proposed by Sutherland (63) in several respects. First,

it can be applied, without special formulation, to any mechanism for which a closed-form solution is available. Sutherland's work deals exclusively with planar four-bar linkages (although extension to other mechanisms is possible). A second difference lies in the method of approximation. Sutherland determines the moving pivot locations which result in the smallest least-squares deviation in the length of the grounded link. The position stacking method, when applied to a planar four-bar linkage, would also include the difference in the locations of the fixed pivot.

The least squares method proposed by Sutherland does offer the possibility of finding an optimal solution in closed form - provided the synthesis equations are linear. Such a solution method has been discussed in greater detail by Gmerek and Matthew (116). Solutions to the position stacking problems presented in this work are found using optimization methods. It should be possible to find closed-form solutions using, for example, the methods presented by Gmerek and Matthew (116), since the synthesis equations are linear in the unknown quantities. However, such a solution probably would not satisfy the constraint conditions and hence would not be an acceptable design.

Mixed sets of exact and approximate positions are believed to have many practical applications (63). The

common transfer operation, where an object is picked up at one position, moved past an intermediate obstacle and placed at another position, is one example. The pick and place positions must be satisfied exactly, whereas intermediate positions only need to be satisfied approximately.

The principal drawback of the position stacking method is that the designer does not have direct control over the error at the approximate positions. The positions are approximated in the sense that the dyad or dyads that generate the approximate positions are similar to the dyad being used, i.e., the one generating the exact positions. It is not possible to evaluate the position error until the dyads have been assembled into a mechanism.

The effectiveness of the position stacking method has been demonstrated in working the examples of this dissertation and also in verifying the numerical example of reference (103). However, more detailed research is needed to determine the potential usefulness of this novel method.

5.3 The Grashof Condition

Grashof analysis, also known as type, limit position or mobility analysis, attempts to determine the relative rotation of links within a given mechanism. A fully

rotating ground-pivoted link is called a crank. A link which oscillates about a fixed pivot between limit positions is called a rocker. The present analysis seeks to determine whether or not the input link of an RCCC mechanism of known dimensions is a crank.

One obvious way to find this out is to perform a kinematic analysis of the relative displacements of the links within the mechanism for a series of closely spaced positions of the input link. A mobility limit position is found when the solution of the loop-closure equation becomes imaginary. Unfortunately, the approach just described involves the independent input of the mechanism. Any optimization based on this approach must necessarily be parametric.

Fortunately, the works of Duffy and Gilmartin (83-85) have eliminated the need for involving the independent input in determining the limit positions of the RCCC mechanism. What follows is a brief summary of portions of their work.

The RCCC mechanism is shown in the notation of Gilmartin and Duffy (84) in figure 5.1. The parameters used to specify it are as follows:

$\underset{\sim}{\hat{s}}_i$ the unit vector along the i^{th} pair axis

$\underset{\sim}{\hat{a}}_{ij}$ the unit vector along the common perpendicular between $\underset{\sim}{\hat{s}}_i$ and $\underset{\sim}{\hat{s}}_j$

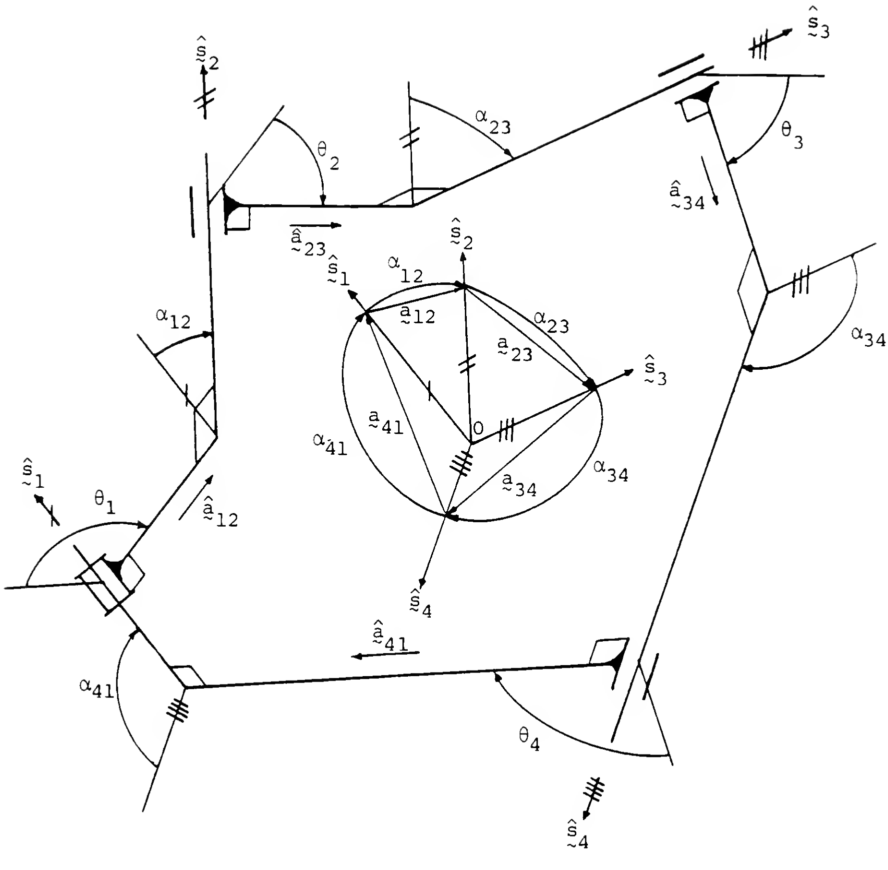


figure 5.1 The RCCC mechanism and its spherical RRRR equivalent

a_{ij} the length of the common perpendicular which is taken as the kinematic link length

S_j the distance between links a_{ij} and a_{jk} along the pair axis $\hat{\underset{\sim}{s}}_j$ (a double subscript such as S_{11} denotes an invariant distance)

α_{ij} the positive angle from $\hat{\underset{\sim}{s}}_i$ to $\hat{\underset{\sim}{s}}_j$ measured clockwise looking in the direction of $\hat{\underset{\sim}{a}}_{ij}$, i.e., in the positive (or right-hand) direction about $\hat{\underset{\sim}{a}}_{ij}$

θ_i the positive angle from $\hat{\underset{\sim}{a}}_{ij}$ to $\hat{\underset{\sim}{a}}_{jk}$ measured clockwise looking in the direction of $\hat{\underset{\sim}{s}}_j$

It can be shown (see for example the text by Duffy (117)) that the spatial RCCC has an equivalent spherical RRRR representation in which all angular relationships within the mechanism are maintained. This equivalent spherical mechanism is also shown in figure 5.1. Notice that the α_{ij}'s become arcs of great circles physically representing the link lengths. Now, since angular relationships are maintained, the spherical mechanism and the spatial mechanism must be of the same type, i.e., crank-rocker, double-crank or double-rocker. The input link rotatability may be determined by first replacing all obtuse link lengths by their supplements so that $0° \leq \alpha_{ij} < 90°$ for all α. Now, the input link, α_{12}, will be a crank if it is the shortest link and if

$$\alpha_{12} + 90° \leq \tfrac{1}{2}(\alpha_{12} + \alpha_{23} + \alpha_{34} + \alpha_{41}) \qquad (5.8)$$

The above expression provides a simple, nonparametric means for determining the input link rotatability in the RCCC mechanism.

5.4 The Branch-Avoidance Condition

The mechanism branching problem was discussed briefly in section 3.2. It occurs when the prescribed positions lie on more than one physical closure of the mechanism. This problem generally renders the solution unsuitable for the task at hand.

Up until now, the only way to avoid this problem in designing the RCCC mechanism was to perform a position analysis on every candidate solution. By checking a mechanism at a large number of positions of the input, the designer could see that the mechanism did or did not pass through the desired positions. This analysis necessarily involved the independent input and was, therefore, parametric optimization.

An easy-to-apply nonparametric method for determining the branching characteristics of the RCCC mechanism will now be developed. Begin by considering the two branches of the planar four-bar mechanism as shown in figure 5.2. It is evident that, for the upper branch

$$\underset{\sim}{a}_{23}^{1} \times \underset{\sim}{a}_{34}^{1} \cdot \underset{\sim}{\hat{s}}_{3} < 0 \qquad (5.9)$$

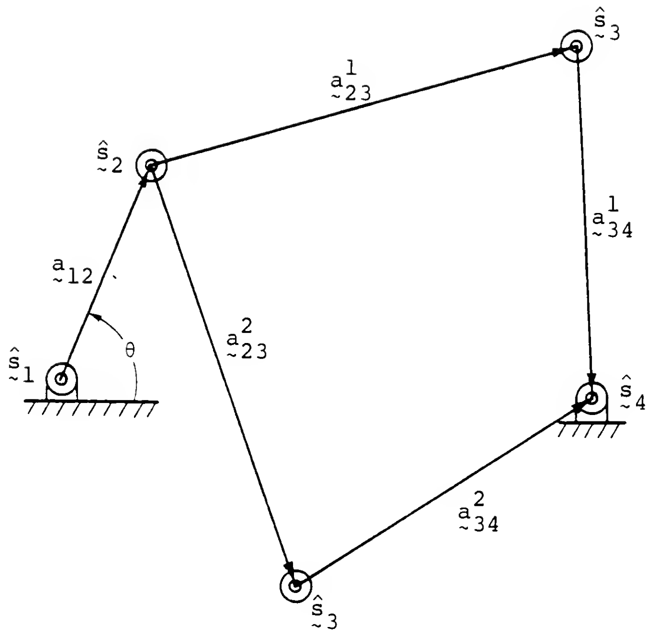


figure 5.2 The two branches of the planar four-bar linkage

and, for the lower branch

$$\underset{\sim}{a}^2_{23} \times \underset{\sim}{a}^2_{34} \cdot \hat{\underset{\sim}{s}}_3 > 0 \tag{5.10}$$

where $\hat{\underset{\sim}{s}}_3$ is the joint axis unit vector pointing out of the plane of the paper and where the superscripts 1 and 2 refer to the first and second position of the mechanism. The mechanism can only move from one branch to the other by passing through a special configuration, where the link vectors $\underset{\sim}{a}_{23}$ and $\underset{\sim}{a}_{34}$ are collinear. The above logic leads to the conclusion that, either

$$\underset{\sim}{a}^j_{23} \times \underset{\sim}{a}^j_{34} \cdot \hat{\underset{\sim}{s}}_3 < 0 \qquad \text{for all } j \tag{5.11}$$

or

$$\underset{\sim}{a}^j_{23} \times \underset{\sim}{a}^j_{34} \cdot \hat{\underset{\sim}{s}}_3 > 0 \qquad \text{for all } j \tag{5.12}$$

if the positions j are to lie on the same branch.

Now returning to the spherical mechanism of figure 5.1, a similar condition can be seen to apply. The vectors $\underset{\sim}{a}_{23}$ and $\underset{\sim}{a}_{34}$ represent chords of the great circles from $\hat{\underset{\sim}{s}}_2$ to $\hat{\underset{\sim}{s}}_3$ and from $\hat{\underset{\sim}{s}}_3$ to $\hat{\underset{\sim}{s}}_4$, respectively. The two branches of the mechanism can be distinguished by the direction of the vector product $\underset{\sim}{a}_{23} \times \underset{\sim}{a}_{34}$. The resulting vector will always be directed toward the inside of the sphere for one branch and toward the outside of the sphere for the other branch. A special configuration

is reached when the resultant vector is tangent to the sphere at joint axis $\hat{\underset{\sim}{s}}_3$. Since $\hat{\underset{\sim}{s}}_3$ is always an outward pointing normal to the sphere, the branch avoidance conditions for the spherical mechanism are the same as for the planar four-bar, i.e., equations 5.11 and 5.12 apply directly.

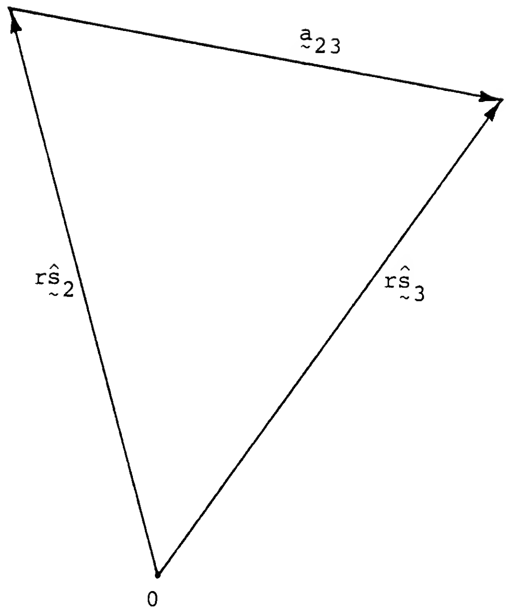
It is convenient to express this result in terms of the joint axes unit vectors only. The vector difference $\hat{\underset{\sim}{s}}_3 - \hat{\underset{\sim}{s}}_2$ produces a vector in the direction of $\underset{\sim}{a}_{23}$, although not necessarily of the same magnitude (see figure 5.3). However, since only the directions of the vectors $\underset{\sim}{a}_{12}$ and $\underset{\sim}{a}_{23}$ are important in equations 5.11 and 5.12, the RRRR spherical mechanism will be free from branching problems if, in all prescribed positions, either

$$(\hat{\underset{\sim}{s}}_3 - \hat{\underset{\sim}{s}}_2) \times (\hat{\underset{\sim}{s}}_4 - \hat{\underset{\sim}{s}}_3) \cdot \hat{\underset{\sim}{s}}_3 = \hat{\underset{\sim}{s}}_4 \times \hat{\underset{\sim}{s}}_2 \cdot \hat{\underset{\sim}{s}}_3 < 0 \qquad (5.13)$$

or

$$(\hat{\underset{\sim}{s}}_3 - \hat{\underset{\sim}{s}}_2) \times (\hat{\underset{\sim}{s}}_4 - \hat{\underset{\sim}{s}}_3) \cdot \hat{\underset{\sim}{s}}_3 = \hat{\underset{\sim}{s}}_4 \times \hat{\underset{\sim}{s}}_2 \cdot \hat{\underset{\sim}{s}}_3 > 0 \qquad (5.14)$$

This constraint condition applies directly to the RCCC mechanism since it is an angular-motion equivalent to the spherical RRRR in terms of relative rotations at each joint. Equations 5.13 and 5.14 are nonparametric



$$\frac{1}{r}\,\underset{\sim}{a}_{23} = \hat{\underset{\sim}{s}}_3 - \hat{\underset{\sim}{s}}_2$$

figure 5.3 The vector chord of the spherical mechanism

constraints which determine the branching characteristics of the RCCC mechanism. Since the branching condition depends only on the directions of the joint axes, it is possible to find a suitable set of joint axes before dimensional synthesis of the mechanism is complete (see Chapter Four, section 4.6).

5.5 The Order Condition

The order problem was introduced briefly in Chapter Three. The order of positions of a moving body is dependent on both the sequence in which the positions are traversed and the sense, or direction, of traversal (88). For example, positions 1,2,3 and 3,2,1 are in the same sequence but have opposite sense. Since the sense can be reversed by reversing the direction of the mechanism's input crank rotation, only the sequence of positions will be of concern here. Notice that only one sequence exists for three positions, i.e., 1,2,3; 2,3,1; 3,1,2; etc., are all in the same sequence. Thus the order problem can only exist when more than three positions are specified.

Most often in mechanism design problems the specified positions naturally occur in a smooth, well-behaved sequence. In such cases it is usually quite easy to satisfy the sequence requirement, since the output of most mechanisms also tends to be smooth

and well-behaved. However, it is still necessary to test if the sequence of output positions is correct. This can be done by finding the rotation of the input link at each of the prescribed positions. If these rotations are in a successively increasing or decreasing sequence, the order condition can be satisfied.

Referring to figure 5.1, θ_1 is the input angle measured from $\hat{\underset{\sim}{a}}_{41}$ to $\hat{\underset{\sim}{a}}_{12}$ in a right-hand sense about $\hat{\underset{\sim}{s}}_1$. If the vector $(\hat{\underset{\sim}{a}}_{12})_j$ represents the jth position of $\hat{\underset{\sim}{a}}_{12}$, then the jth position of θ, θ_j, is given by

$$\theta_j = \arccos\{(\hat{\underset{\sim}{a}}_{12})_j \cdot \hat{\underset{\sim}{a}}_{41}\}$$

$$\text{if } (\hat{\underset{\sim}{a}}_{12})_j \cdot \hat{\underset{\sim}{a}}_{41} \times \hat{\underset{\sim}{s}}_1 \geq 0 \tag{5.15}$$

and

$$\theta_j = 2\pi - \arccos\{(\hat{\underset{\sim}{a}}_{12})_j \cdot \hat{\underset{\sim}{a}}_{41})\}$$

$$\text{if } (\hat{\underset{\sim}{a}}_{12})_j \cdot \hat{\underset{\sim}{a}}_{41} \times \hat{\underset{\sim}{s}}_1 < 0 \tag{5.16}$$

Let $\theta_1, \theta_2, \theta_3$ and θ_4 be the values of θ at four prescribed positions. The sequence of these four positions will be correct if, after arranging the four angles in a series from the smallest to the largest, any of the following eight combinations result:

$$\begin{array}{ll} \theta_1, \theta_2, \theta_3, \theta_4 & \theta_4, \theta_3, \theta_2, \theta_1 \\ \theta_2, \theta_3, \theta_4, \theta_1 & \theta_3, \theta_2, \theta_1, \theta_4 \\ \theta_3, \theta_4, \theta_1, \theta_2 & \theta_2, \theta_1, \theta_4, \theta_3 \\ \theta_4, \theta_1, \theta_2, \theta_3 & \theta_1, \theta_4, \theta_3, \theta_2 \end{array} \qquad (5.17)$$

Notice that the left-hand set above is in the correct order, i.e., the positions are traversed in the correct sequence and sense. The right-hand set will be in the correct order if the direction of input crank rotation is reversed.

The procedure just described is a nonparametric method for determining if the prescribed positions are satisfied in the correct order.

5.6 Fixed-Pivot Location and Link-Length Ratio Conditions

The fixed-pivot locations are the points in the global reference system where the mechanism is physically connected to the fixed machine frame. For both the RC and the CC dyad, such fixed points are located by the vector, $\underset{\sim}{a}_0$, as shown in figure 4.9. Restrictions on the value of $\underset{\sim}{a}_0$ will be strongly dependent on the specific problem at hand. However, these constraints will generally be quite easy to apply. For example, the constraint

$$a_{0x} < 0 \qquad (5.18)$$

could be used to restrict the location of the fixed pivot to the left of the plane defined by the Y and Z axes.

Link-length ratio conditions will also be quite easy to apply in practice. Referring to figure 5.1, a_{12}, a_{23}, a_{34} and a_{41} are the kinematic link lengths of the RCCC mechanism. One possible requirement might be that the ratio of the longest to the shortest link length be less than ten, or

$$\frac{a_{longest}}{a_{shortest}} - 10 < 0 \tag{5.19}$$

It should be noted that the RCCC mechanism may be viable even when $a_{shortest}$ is equal to zero. It may therefore be desireable to restrict only the maximum allowable link length. Both of the above-mentioned constraints will generally be nonparametric.

5.7 The Objective Function

As discussed in Chapter Two, the objective function serves as a measure of the relative merits of a particular design. Since each of the requirements discussed in this chapter must be included in determining the relative merits of a particular design, the objective function (O.F.) will be of the following form:

$$O.F. = M.E.T. + C.R.T. + B.A.T. + O.P.T. + F.P.L.T. + L.L.R.T. \qquad (5.20)$$

where

M.E.T. = Motion Error Term

C.R.T. = Crank-Rotatability Term

B.A.T. = Branch-Avoidance Term

O.P.T. = Order-of-Positions Term

F.P.L.T. = Fixed-Pivot Location Term

L.L.R.T. = Link-Length Ratio Term

Motion Error Term. The first term in the above expression, M.E.T., will result from application of equation 5.7 to the CC dyad and the RC dyad. The total error will be defined as the sum of the absolute values of the resulting errors and will provide a direct measure of the success in approximating the fourth prescribed position. The remaining terms are all constraint conditions which will be incorporated using penalty function methods. In many cases, the most difficult problem is finding a suitable measure of the amount by which a constraint is violated. A brief discussion of each constraint condition follows:

Crank-Rotatability Term. Part of this term follows directly from equation 5.8 and the penalty function methods of section 2.4.2. However, an additional constraint is needed to ensure that the input link, α_{12}, is the shortest link. This can be accomplished by assessing the following penalty when α_{12} is not the shortest link:

$\Delta\alpha$ = largest of

$$\{\langle\alpha_{12} - \alpha_{23}\rangle, \langle\alpha_{12} - \alpha_{34}\rangle, \langle\alpha_{12} - \alpha_{41}\rangle\} \tag{5.21}$$

where the angular brackets indicate the singularity function as defined by equation 2.30. It can be seen that $\Delta\alpha$ will equal zero when α_{12} is the shortest link. Otherwise $\Delta\alpha$ will be a positive number which indicates the amount by which α_{12} differs from the shortest link. With this, the crank rotatability term becomes

$$\text{C.R.T.} = r\{\langle\alpha_{12} + 90° - \tfrac{1}{2}(\alpha_{12} + \alpha_{23} + \alpha_{34} + \alpha_{41})\rangle + \Delta\alpha\} \tag{5.22}$$

where r is the penalty parameter defined in section 2.4.2. Equation 5.22 will equal zero when the input link of the RCCC mechanism is a crank. Any result greater than zero serves as an indication of the amount by which the mechanism fails to meet this requirement.

<u>Branch-Avoidance Term</u>. The branch-avoidance conditions are given by equations 5.13 and 5.14. For convenience, the function F_b is defined to be equal to the left-hand side of these equations, i.e.

$$F_b = (\hat{\underset{\sim}{s}}_3 - \hat{\underset{\sim}{s}}_2) \times (\hat{\underset{\sim}{s}}_4 - \hat{\underset{\sim}{s}}_3) \cdot \hat{\underset{\sim}{s}}_3 \tag{5.23}$$

so that F_b must be either greater than zero or less than zero in all positions if branching problems are to be avoided. When branching does occur, it will be necessary

to have some measure of the magnitude of the problem. One such measure is given by

$$\text{B.A.T.} = r\{\text{smallest of } (\Sigma F_{bg}, |\Sigma F_{b\ell}|)\} \tag{5.24}$$

where ΣF_{bg} is the sum of the F_b's greater than zero at the prescribed positions and $|\Sigma F_{b\ell}|$ is the absolute value of the sum of all F_b's less than zero at the prescribed positions. When all F_b's are of the same sign, B.A.T. will equal zero. Notice that this term will only be an approximation for positions which are approximately satisfied.

Order-of-Positions Term. Equation 5.17 gives all possible combinations of the four values of θ which are in the correct sequence. Again the problem arises of what penalty to assess when this constraint is violated. Regardless of the arrangement of the four θ values, a correct sequence can always be obtained by interchanging two values of θ. However, two possibilities always exist for this interchange. Therefore, it is necessary to scan the possible choices to find the most closely spaced pair of angles which can be interchanged to give the correct sequence. These angles will be called θ_a and θ_b. Now the difference between these two angles can serve as a measure of the error in satisfying the order condition, i.e.

$$\text{O.P.T.} = r|\theta_a - \theta_b| \tag{5.25}$$

When the sequence is correct, θ_a and θ_b will be set equal to zero.

<u>Fixed-Pivot Location Term</u>. Application of this constraint condition will usually be simple and direct. The components of the fixed pivot location vector $\underset{\sim}{a}_0$ will most often be restricted to lie within a specified range, for example

$$x_a < a_{0x} < x_b \tag{5.26}$$

This contributes the following term to the objective function

$$\text{F.P.L.T.} = r\{\left\langle x_a - a_{0x} \right\rangle + \left\langle a_{0x} - x_b \right\rangle\} \tag{5.27}$$

where, again, r is the penalty parameter.

<u>Link-Length Ratio Term</u>. Application of this constraint will also usually be simple and direct. For example, equation 5.19 would result in the following penalty term:

$$\text{L.L.R.T.} = r\left\langle (a_{longest}/a_{shortest}) - 10 \right\rangle \tag{5.28}$$

It can be seen from the foregoing discussion that formulating the objective function is far from an exact science. This work represents the first known attempt

to apply many of the nonparametric constraint conditions discussed in this chapter to the optimization of the RCCC mechanism. As a result, no basis exists for comparing the formulations presented here. Future research will undoubtedly lead to improved and more sophisticated methods for assessing penalties when constraints are violated.

5.8 Numerical Example

The following positions and orientations of the spatial moving body are specified:

$$\underset{\sim}{o}_1 = \{0\hat{\underset{\sim}{i}} + 0\hat{\underset{\sim}{j}} + 0\hat{\underset{\sim}{k}}\}^T$$

$$\underset{\sim}{o}_2 = \{4\hat{\underset{\sim}{i}} + 2\hat{\underset{\sim}{j}} + 0\hat{\underset{\sim}{k}}\}^T$$

$$\underset{\sim}{o}_3 = \{8\hat{\underset{\sim}{i}} + 2\hat{\underset{\sim}{j}} + 3\hat{\underset{\sim}{k}}\}^T$$

$$\underset{\sim}{o}_4 = \{10\hat{\underset{\sim}{i}} + 3\hat{\underset{\sim}{j}} + 5\hat{\underset{\sim}{k}}\}^T$$

$$R_2 = \begin{bmatrix} .966 & -.258 & .008 \\ .258 & .962 & -.087 \\ .015 & .086 & .996 \end{bmatrix}$$

$$R_3 = \begin{bmatrix} .868 & -.494 & .045 \\ .489 & .834 & -.255 \\ .089 & .243 & .966 \end{bmatrix}$$

$$R_4 = \begin{bmatrix} .824 & -.564 & .059 \\ .548 & .765 & .337 \\ .145 & .310 & .940 \end{bmatrix}$$

Recall from section 4.4 that $[R_1]$ is assumed to be a 3x3 identity matrix.

Problem Statement. Design an RCCC mechanism which exactly satisfies the first three of the above positions and orientations. The fourth position and orientation are to be satisfied in an approximate sense using the position-stacking method. The solution mechanism is required to be of the crank-rocker type, free from branching problems. The fixed pivots should be located within one-hundred units of the global reference origin and the ratio of the longest to the shortest link length should not exceed ten units.

Solution. Solution mechanisms are obtained by minimizing the objective function of equation 5.20 using the Hooke and Jeeve's optimization method. Appendix 1 provides a complete listing of all the APL-language computer programs used in solving this problem along with a sample run of the program package.

Figure 5.4 shows the RCCC mechanism and associated design parameters. Notice that the fixed and moving coordinate systems are coincident in the initial position. Tables 5.1 and 5.2 give the results of two separate program runs which converged to feasible solutions.

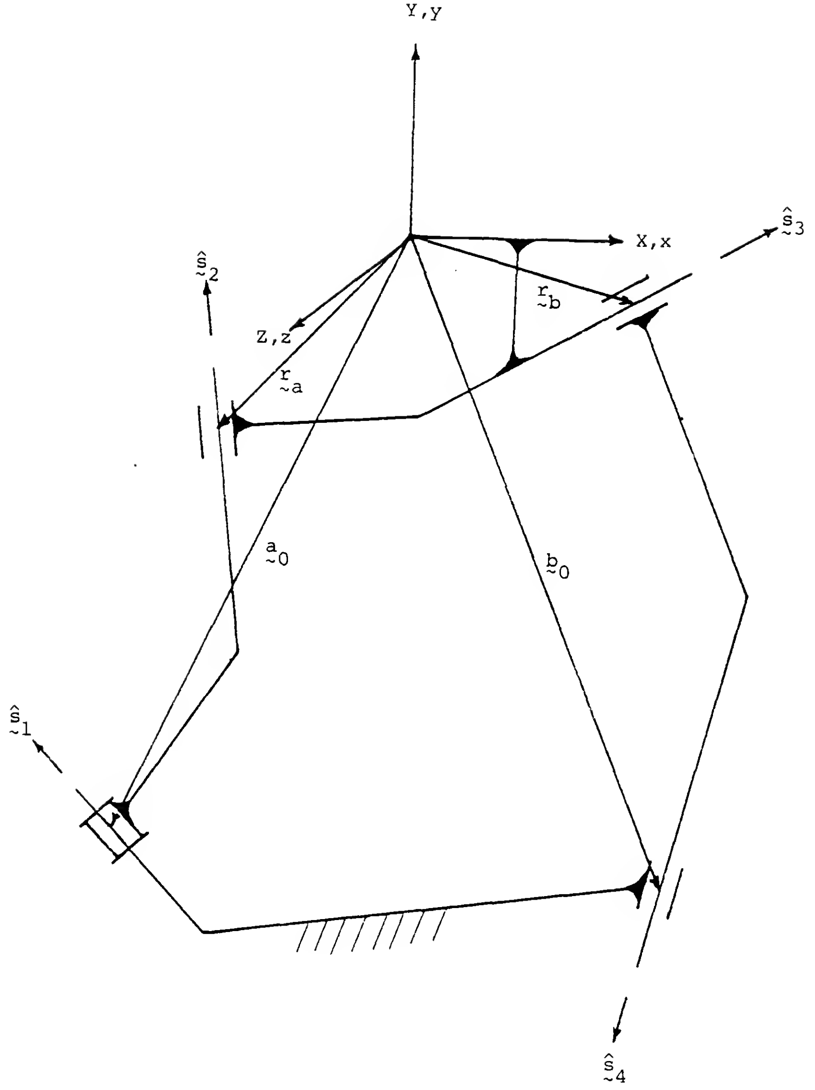


figure 5.4 The RCCC mechanism and associated design parameters

Table 5.1

RCCC Mechanism Optimization - Example 1

Assumed quantities (refer to figure 5.4)

$\hat{\underset{\sim}{s}}_1 = \{0\hat{\underset{\sim}{i}} + 1\hat{\underset{\sim}{j}} + 0\hat{\underset{\sim}{k}}\}$ $\quad$ $S1_2 = S1_3 = 3$

$\hat{\underset{\sim}{s}}_4 = \{.981\hat{\underset{\sim}{i}} + .173\hat{\underset{\sim}{j}} - .086\hat{\underset{\sim}{k}}\}$ $\quad$ (displacements along $\hat{\underset{\sim}{s}}_4$)

After 35 iterations of the APL program HOOKE and a total of 601 evaluations of the objective function, all constraints were satisfied and the solution had converged to the results given below.

		$\hat{\underset{\sim}{i}}$	$\hat{\underset{\sim}{j}}$	$\hat{\underset{\sim}{k}}$	
$\underset{\sim}{a}_0$	=	-22.634	13.635	-9.229	
$\hat{\underset{\sim}{s}}_2$	=	.281	-.363	.889	
$\underset{\sim}{r}_a$	=	-18.595	14.848	-10.012	
$\underset{\sim}{b}_0$	=	-12.188	29.798	3.518	mechanism satisfying positions 1,2 and 3
$\hat{\underset{\sim}{s}}_3$	=	-.307	-.201	.930	
$\underset{\sim}{r}_b$	=	- 7.233	17.979	2.594	

		$\hat{\underset{\sim}{i}}$	$\hat{\underset{\sim}{j}}$	$\hat{\underset{\sim}{k}}$	
$\underset{\sim}{a}_0$	=	-23.349	13.601	-9.864	
$\hat{\underset{\sim}{s}}_2$	=	.281	-.363	.889	
$\underset{\sim}{r}_a$	=	-19.142	14.865	-10.680	mechanism satisfying positions 1,2 and 4
$\underset{\sim}{b}_0$	=	-5.656	16.266	2.341	
$\hat{\underset{\sim}{s}}_3$	=	-.469	-.241	.850	
$\underset{\sim}{r}_b$	=	-6.859	19.197	2.510	

For both mechanisms

$\hat{\underset{\sim}{s}}_1 = \{.314\hat{\underset{\sim}{i}} - .542\hat{\underset{\sim}{j}} + .780\hat{\underset{\sim}{k}}\}$ $\quad$ $S1_2 = -.089$ $\quad$ $S1_3 = .781$

$\hat{\underset{\sim}{s}}_4 = \{.869\hat{\underset{\sim}{i}} + .336\hat{\underset{\sim}{j}} + .363\hat{\underset{\sim}{k}}\}$

Table 5.2

RCCC Mechanism Optimization - Example 2

Assumed quantities (refer to figure 5.4)

$\underset{\sim}{\hat{s}}_1 = \{0\underset{\sim}{\hat{i}} + 1\underset{\sim}{\hat{j}} + 0\underset{\sim}{\hat{k}}\}$ $\qquad S1_2 = S1_3 = -3$

$\underset{\sim}{\hat{s}}_4 = \{0\underset{\sim}{\hat{i}} + 1\underset{\sim}{\hat{j}} + 0\underset{\sim}{\hat{k}}\}$ $\qquad$ (displacements along $\underset{\sim}{\hat{s}}_4$)

After 25 iterations of the APL function HOOKE and a total of 541 evaluations of the objective function, all constraints were satisfied and the solution had converged to the results given below.

	$\underset{\sim}{\hat{i}}$	$\underset{\sim}{\hat{j}}$	$\underset{\sim}{\hat{k}}$	
$\underset{\sim}{a}_0$ =	-15.271	6.477	-62.412	
$\underset{\sim}{\hat{s}}_2$ =	.725	.457	.515	
$\underset{\sim}{r}_a$ =	-12.704	2.653	-62.633	mechanism satisfying positions 1,2 and 3
$\underset{\sim}{b}_0$ =	-10.495	10.246	-20.034	
$\underset{\sim}{\hat{s}}_3$ =	.137	-.061	.989	
$\underset{\sim}{r}_b$ =	-12.092	14.134	-19.574	

	$\underset{\sim}{\hat{i}}$	$\underset{\sim}{\hat{j}}$	$\underset{\sim}{\hat{k}}$	
$\underset{\sim}{a}_0$ =	-10.833	7.950	-42.64	
$\underset{\sim}{\hat{s}}_2$ =	.720	.447	.529	
$\underset{\sim}{r}_a$ =	-9.041	5.229	-42.771	mechanism satisfying positions 1,2 and 4
$\underset{\sim}{b}_0$ =	-13.179	22.411	-12.152	
$\underset{\sim}{\hat{s}}_3$ =	.130	-.063	.989	
$\underset{\sim}{r}_b$ =	-10.393	15.642	-12.947	

For both mechanisms

$\underset{\sim}{\hat{s}}_1 = \{.564\underset{\sim}{\hat{i}} + .335\underset{\sim}{\hat{j}} + .755\underset{\sim}{\hat{k}}\}$ $\qquad S1_2 - .375 \qquad S1_3 - -.019$

$\underset{\sim}{\hat{s}}_4 = \{.668\underset{\sim}{\hat{i}} + .352\underset{\sim}{\hat{j}} + -.656\underset{\sim}{\hat{k}}\}$

CHAPTER 6

OPTIMIZATION OF THE RSSR-SC AND RSSR-SS MECHANISMS

6.1 Problem Formulation

Chapter Five presented what is believed to be the most complete discourse to date on designing the RCCC mechanism for spatial rigid-body guidance. However, several important problems remain to be solved in the design of this mechanism. First, nonparametric conditions are needed to determine its transmission characteristics. These conditions must include consideration of the forces which cause binding of the cylindric joints along their axes. Second, the problem of cylindric joint axes becoming collinear during the motion cycle must be addressed. When this occurs, the displacements along these joint axes go to infinity, and the mechanism becomes useless. Until these problems are solved, the potential uses of the RCCC mechanism will remain quite limited.

This chapter discusses the design of the five-link RSSR-SC and RSSR-SS mechanisms for spatial rigid-body guidance. These mechanisms are shown in figures 6.1 and 6.2. As a result of the problems discussed above, the

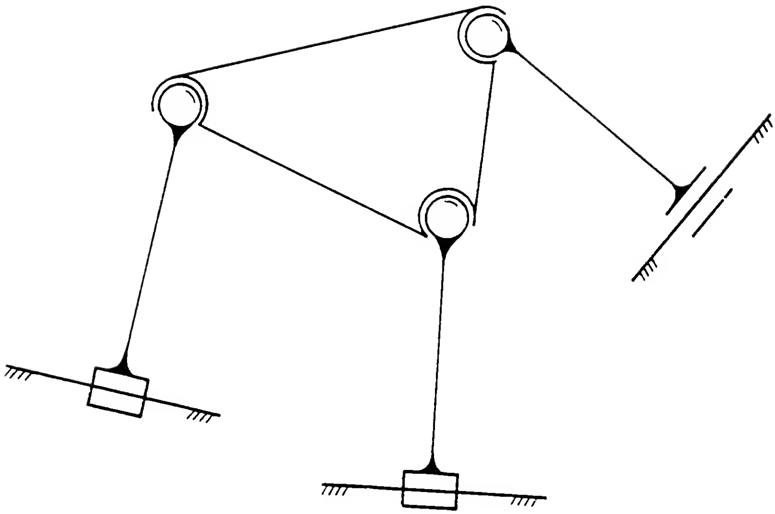


figure 6.1 The RSSR-SC mechanism

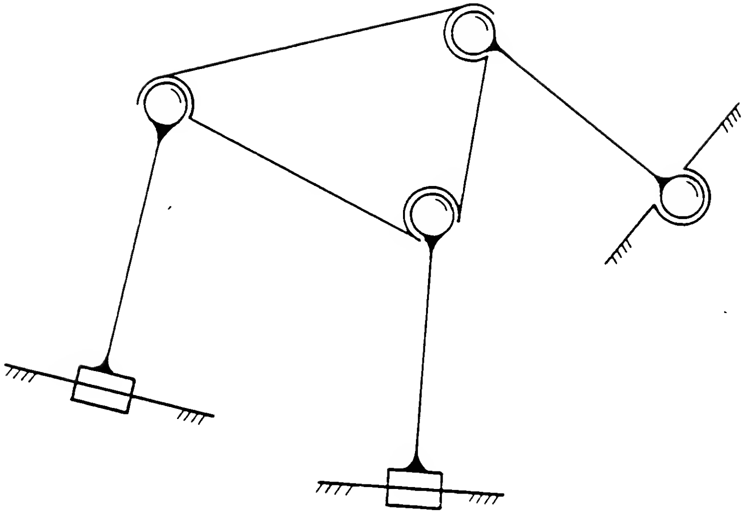


figure 6.2 The RSSR-SS mechanism

author believes these mechanisms are currently more useful than the RCCC mechanism. Also, since the five-link mechanisms presented in this chapter have three grounded joints, they will tend to be more stable in holding a certain position than the RCCC mechanism, which has only two grounded joints.

The objective of this chapter is to develop procedures for designing RSSR-SC and RSSR-SS mechanisms for three positions of rigid-body guidance. Constraints dealing with branch avoidance, Grashof type, transmission characteristics, link-length ratios and fixed pivot locations will be developed and applied. Also discussed are the use of the position stacking method and the order problem when more than three positions are specified.

6.2 Method of Design

The RSSR-SC and RSSR-SS mechanisms may be synthesized for three positions of rigid-body guidance using the methods of Chapter Four. This requires finding two RS dyads and either a CS or an SS dyad whose floating links satisfy the specified motion. The floating links of these dyads are then connected to form a single-degree-of-freedom mechanism.[1]

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[1]The RSSR-SS mechanism possesses an additional idle degree of freedom which is the free rotation of the grounded link of the SS dyad about its own axis.

Ideally, the remaining requirements, such as branch-avoidance, Grashof type, etc., could be formulated non-parametrically for each mechanism, as was done for the RCCC mechanism in the previous chapter. Unfortunately, the relative complexity of the RSSR-SC and the RSSR-SS mechanisms renders this approach impractical, at least at the present time. An alternate approach, based on the design formulation of the RSSR mechanism is now proposed.

The RS dyad may be thought of as a special case of both the CS and SS dyads. The RS dyad is obtained from the CS dyad by removing the axial translation, and obtained from the SS dyad by removing two rotational freedoms from the grounded S joint. Three RS dyads may be synthesized for three positions of rigid-body guidance and assembled into an RSSR-SR structure, as shown in figure 6.3. This structure can be assembled at the prescribed positions, but cannot move between them. However, replacing any of the grounded revolute joints by either a cylindric joint or a spheric joint results in viable, single-degree-of-freedom mechanisms capable of generating the prescribed motion. By designing the RSSR-SC and RSSR-SS mechanisms in this way, they may be considered to be a pair of RSSR mechanisms with one dyad in common, at least at the

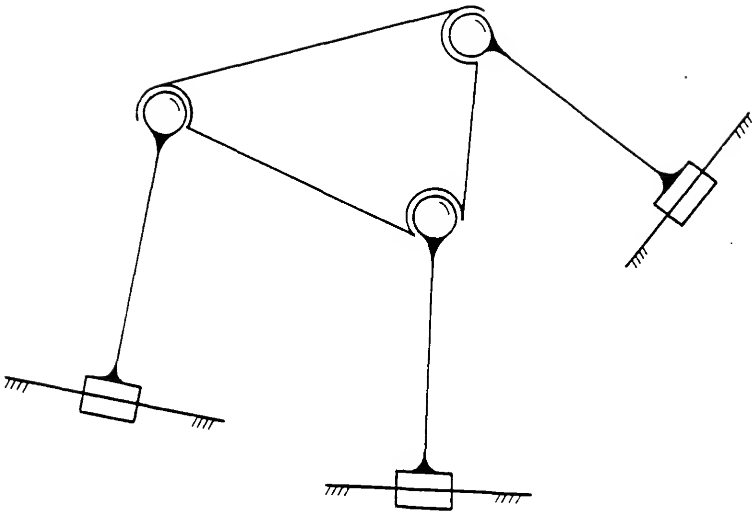

figure 6.3 The RSSR-SR structure

precision positions. This will greatly facilitate finding nonparametric conditions for the various design requirements, although some of these conditions will be approximations. For the remainder of this chapter, the design of the RSSR and the RSSR-SR mechanism will be used as an intermediate step in the design of the RSSR-SC and the RSSR-SS mechanisms. However, the constraint conditions developed for the RSSR mechanism are extremely important in their own right, since this is one of the most often used spatial mechanisms for function generation.

6.3 The Branch-Avoidance Condition

The mechanism branching problem has been discussed in sections 3.2 and 5.3. It is evident that kinematic position analysis is often too inefficient to incorporate in an optimization loop.

Fortunately, the recent works of Gupta and Tinubu (92) and Sandor and Zhuang (93,94) have eliminated the need for this time-consuming analysis by devising non-parametric tests to determine if the branching problem is present in the RSSR mechanism. Since the method of Sandor and Zhuang (93,94) applies directly to motion-generating mechanisms, and since it can be applied to a variety of other mechanisms as well, it will be demonstrated here. It is important to realize that the present analysis applies exactly to the RSSR-SC and

RSSR-SS mechanisms. This is true because the branching conditions are applied only at the precision positions, where the configuration of the RSSR-SR mechanism is the same as these other mechanisms.

Figure 6.4 shows a schematic representation of the RSSR mechanism. The two branches can be determined by holding input link A_0A fixed and finding the intersections of the loci of spheric joint B about spheric joint A and of spheric joint B about revolute joint B_0. These loci are a sphere and a circle, respectively, as shown in figure 6.5.

It has been shown that when the two branches coincide, links AB, B_0B and axis $\hat{\underset{\sim}{s}}_2$ are coplanar (93); this is a so-called special configuration of the mechanism. When the branches are separated, the two intersections will be located on opposite sides of this plane, and cannot change sides without passing through a special configuration of the mechanism. Using the subscript j to denote the prescribed positions of the mechanism, the above logic leads to the conclusion that either

$$\overrightarrow{A_jB_j} \cdot \hat{\underset{\sim}{s}}_2 \times \overrightarrow{B_0B_j} > 0 \qquad \text{for all } j \qquad (6.1)$$

or

$$\overrightarrow{A_jB_j} \cdot \hat{\underset{\sim}{s}}_2 \times \overrightarrow{B_0B_j} < 0 \qquad \text{for all } j \qquad (6.2)$$

if the positions j are to lie on one branch only.

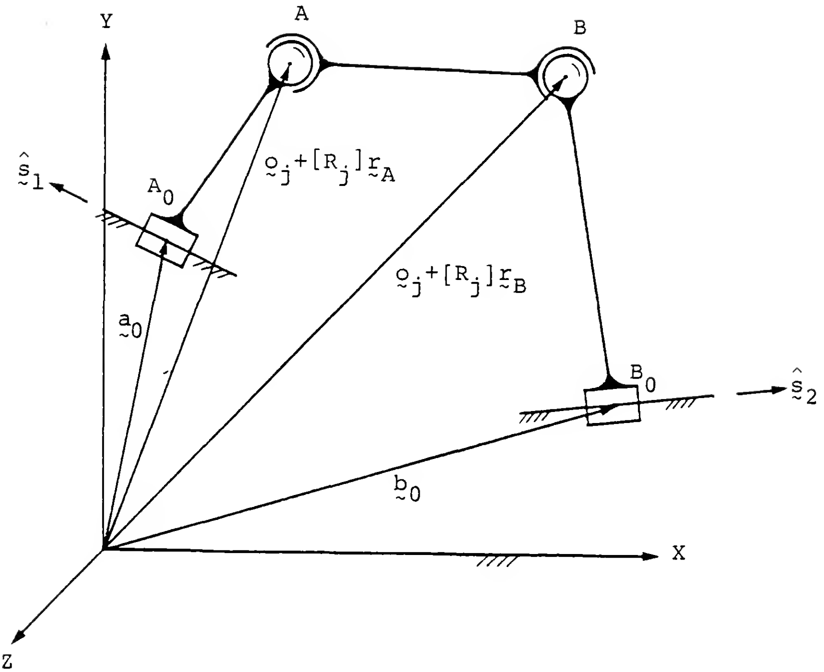

figure 6.4 The RSSR mechanism

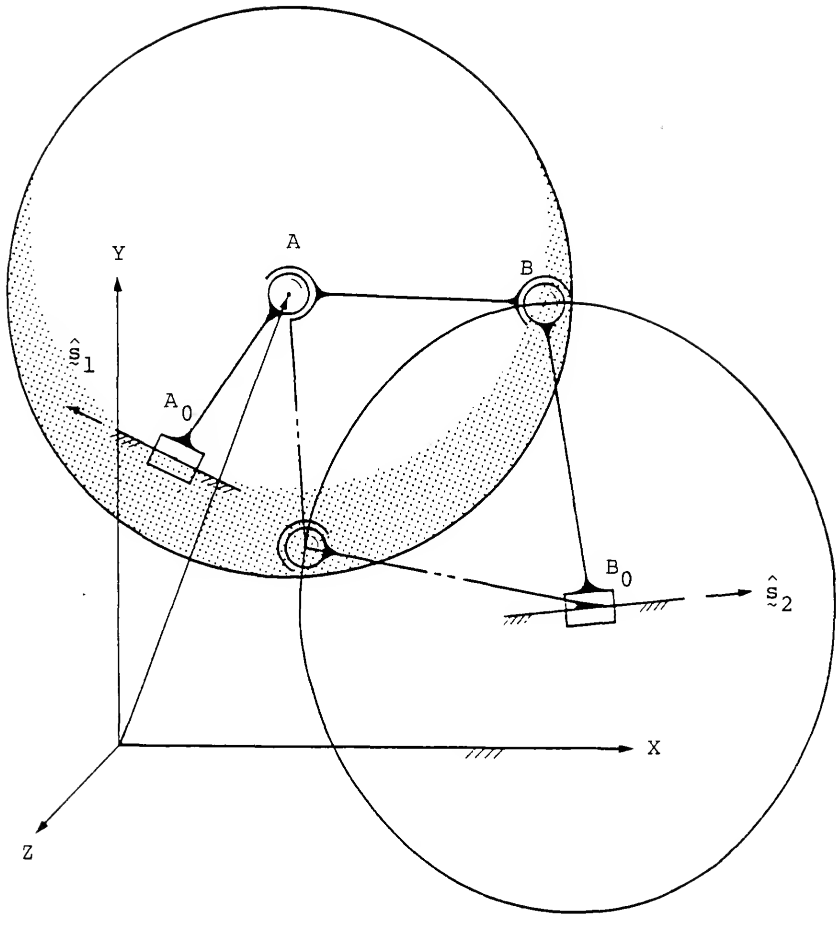

figure 6.5 The two branches of the RSSR mechanism

It should be noted that the vector $\overrightarrow{A_jB_j}$ is simply the vector difference in the locations of the two spheric joints for a pair of independently synthesized RS dyads. For example, if $\underset{\sim}{r}_A$ and $\underset{\sim}{r}_B$ are used to denote the location of the spheric joint relative to the moving coordinate system for dyads A and B, then

$$\begin{aligned} \overrightarrow{A_jB_j} &= \underset{\sim}{o}_j + [R_j]\underset{\sim}{r}_B - (\underset{\sim}{o}_j + [R_j]\underset{\sim}{r}_A) \\ &= [R_j]\underset{\sim}{r}_B - [R_j]\underset{\sim}{r}_A \\ &= [R_j](\underset{\sim}{r}_B - \underset{\sim}{r}_A) \end{aligned} \qquad (6.3)$$

The vector representing the output link, $\overrightarrow{B_0B_j}$, is given by

$$\overrightarrow{B_0B_j} = \underset{\sim}{o}_j + ([R_j]\underset{\sim}{r}_B) - \underset{\sim}{b}_0 \qquad (6.4)$$

where $\underset{\sim}{b}_0$ locates the grounded revolute joint center of dyad B. It can be seen that constraint conditions 6.1 and 6.2 are simple, nonparametric expressions that can be used to identify RSSR mechanisms without branching problems.

As has already been discussed, the RSSR-SR mechanism can be considered to be two RSSR mechanisms with one dyad in common. The forgoing analysis may therefore be applied to the two RSSR mechanisms to determine the branching characteristics of the overconstrained RSSR-SR mechanism, and hence, the RSSR-SC and RSSR-SS mechanisms.

6.4 The Grashof Condition

Sections 3.2 and 5.3 presented brief discussions on the nature and importance of the Grashof condition. The goal of this section is to determine input-link rotatability conditions for the RSSR-SC and the RSSR-SS mechanisms. Unfortunately, these conditions will not be exact, since they will be based on the input-link rotatability conditions for an independent pair of RSSR mechanisms. When the coupler links of the RSSR mechanisms are joined, and one of the revolute joints is replaced by a spheric or a cylindric joint, the motion of that joint will no longer be pure rotation. Because of this, the Grashof condition for the RSSC or RSSS loop of the mechanism will be an approximation. While this analysis will not guarantee the correct mechanism type, it is likely to yield an acceptable solution in far fewer iterations of position analysis than would otherwise be required. Additionally, it should be noted that either revolute joint which is not the input may be replaced by a cylindric or spheric joint. It is quite possible that one case will yield an acceptable solution when the other case does not. Therefore, both possibilities should be explored when searching for an acceptable design.

Several authors have devised nonparametric methods to identify crank rotatability of the RSSR mechanism

(86,118,119,120). Adopting the notation of Nolle (119), the RSSR mechanism is shown in figure 6.6. The input link axis is coincident with the Z axis, and the origin of the coordinate system is located at the intersection of this axis and the common perpendicular to the two revolute joint axes. The input angle, θ, and the output angle, ϕ, are measured from lines parallel to the X axis in a right-hand sense about the respective joint axes. In terms of the notation of Chapter Four, the link lengths a,b and c are simply the magnitudes of the previously determined vectors representing these links. The length of the common normal, e, is given by

$$e = (\underset{\sim}{b}_0 - \underset{\sim}{a}_0) \cdot ((\underset{\sim}{\hat{s}}_1 \times \underset{\sim}{\hat{s}}_2) \div |\underset{\sim}{\hat{s}}_1 \times \underset{\sim}{\hat{s}}_2|) \tag{6.5}$$

where $\underset{\sim}{a}_0$, $\underset{\sim}{b}_0$, $\underset{\sim}{\hat{s}}_1$ and $\underset{\sim}{\hat{s}}_2$ are the revolute joint locations and the joint axis unit vectors, as shown in figure 6.4. The scalar quantity, f, can be determined from

$$e(\underset{\sim}{\hat{s}}_1 \times \underset{\sim}{\hat{s}}_2) + g\underset{\sim}{\hat{s}}_2 + (\underset{\sim}{a}_0 - \underset{\sim}{b}_0) = f\underset{\sim}{\hat{s}}_1 \tag{6.6}$$

Cross-multiplying both sides by $\underset{\sim}{\hat{s}}_2$ eliminates the term containing g and gives

$$e(\underset{\sim}{\hat{s}}_1 \times \underset{\sim}{\hat{s}}_2) \times \underset{\sim}{\hat{s}}_2 + \cancelto{0}{g\underset{\sim}{\hat{s}}_2 \times \underset{\sim}{\hat{s}}_2} + (\underset{\sim}{a}_0 - \underset{\sim}{b}_0) \times \underset{\sim}{\hat{s}}_2 = f\underset{\sim}{\hat{s}}_1 \times \underset{\sim}{\hat{s}}_2 \tag{6.7}$$

Finally, taking the scalar product of both sides of equation 6.7 with the vector $(\underset{\sim}{\hat{s}}_1 \times \underset{\sim}{\hat{s}}_2)$ and solving for f gives

$$f = \frac{(\underset{\sim}{a}_0 - \underset{\sim}{b}_0) \cdot \underset{\sim}{\hat{s}}_2 \times (\underset{\sim}{\hat{s}}_1 \times \underset{\sim}{\hat{s}}_2)}{(\underset{\sim}{\hat{s}}_1 \times \underset{\sim}{\hat{s}}_2) \cdot (\underset{\sim}{\hat{s}}_1 \times \underset{\sim}{\hat{s}}_2)} \tag{6.8}$$

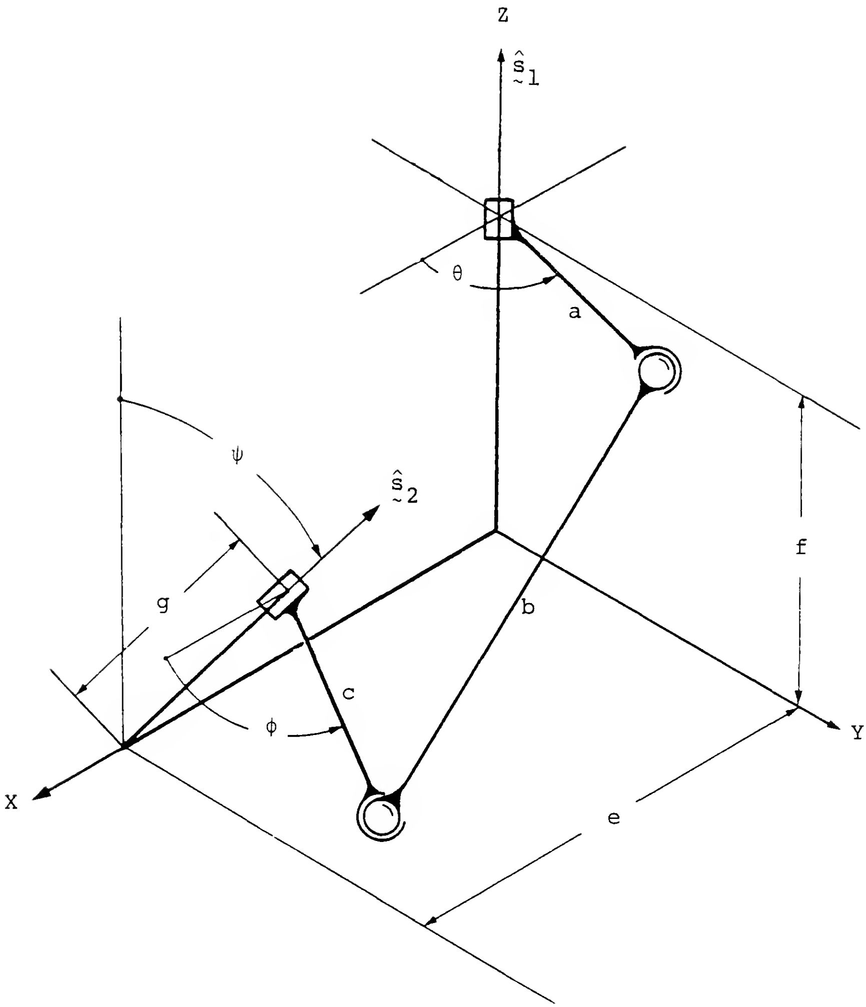

figure 6.6 The RSSR mechanism in the coordinate system of Nolle (119)

Similarly, g can be shown to be

$$g = \frac{(\underset{\sim}{a}_0 - \underset{\sim}{b}_0) \times \hat{\underset{\sim}{s}}_1 \cdot (\hat{\underset{\sim}{s}}_1 \times \hat{\underset{\sim}{s}}_2)}{(\hat{\underset{\sim}{s}}_1 \times \hat{\underset{\sim}{s}}_2) \cdot (\hat{\underset{\sim}{s}}_1 \times \hat{\underset{\sim}{s}}_2)} \tag{6.9}$$

The angle between the revolute joint axes, ψ, is given by

$$\psi = \arccos\ (\hat{\underset{\sim}{s}}_1 \cdot \hat{\underset{\sim}{s}}_2) \tag{6.10}$$

It should be noted that negative values of ψ need not be considered, because the Grashof type is a symmetric property about the value $\psi = 0$.

The relationship between the input angle, θ, and the output angle, ϕ, has been shown to be (118)

$$\tan \phi/2 = \frac{1 \pm [1 + F_2^2(\theta) - F_1^2(\theta)]^{\frac{1}{2}}}{F_1(\theta) - F_2(\theta)} \tag{6.11}$$

where

$$F_1(\theta) = \frac{k_1 + k_2\cos\theta + k_3\sin\theta}{k_6 + k_7\sin\theta} \tag{6.12}$$

$$F_2(\theta) = \frac{k_4 + k_5\cos\theta}{k_6 + k_7\sin\theta} \tag{6.13}$$

and

$$k_1 = fg \cos\psi - \tfrac{1}{2}(f^2 + a^2 + e^2 + g^2 + c^2 - b^2) \qquad (6.14)$$

$$k_2 = ae \qquad (6.15)$$

$$k_3 = ag \sin\psi \qquad (6.16)$$

$$k_4 = -ce \qquad (6.17)$$

$$k_5 = ca \qquad (6.18)$$

$$k_6 = cf \sin\psi \qquad (6.19)$$

$$k_7 = -ca \cos\psi \qquad (6.20)$$

The two roots of equation 6.11 correspond to the two possible values of the output, ϕ, for any given value of the input, θ. However, when the input link reaches a limit position, the two values of ϕ become the same (118). For this to happen, the quantity within the radical of equation 6.11 must equal zero, i.e.,

$$1 + F_2^2(\theta) + F_1^2(\theta) = 0 \qquad (6.21)$$

Since $F_1^2(\theta)$ and $F_2^2(\theta)$ are quadratic in $\cos\theta$ and $\sin\theta$, the following substitutions are made:

$$\cos\theta = x \qquad (6.22)$$

$$\sin\theta = y \qquad (6.23)$$

$$y^2 = 1 - x^2 \qquad (6.24)$$

With these, equation 6.21 may be rewritten as

$$x^4 + Bx^3 + Cx^2 + Dx + E = 0 \tag{6.25}$$

where

$$B = (2K_2K_5 + 2K_3K_4)/(K_4^2 + K_5^2) \tag{6.26}$$

$$C = (K_2^2 + K_3^2 - K_4^2 + 2K_1K_5)/(K_4^2 + K_5^2) \tag{6.27}$$

$$D = (2K_1K_5 - 2K_3K_4)/(K_4^2 + K_5^2) \tag{6.28}$$

$$E = (K_1^2 - K_3^2)/(K_4^2 + K_5^2) \tag{6.29}$$

and

$$K_1 = k_4^2 + k_6^2 + k_7^2 - k_1^2 - k_3^2 \tag{6.30}$$

$$K_2 = 2k_4k_5 - 2k_1k_2 \tag{6.31}$$

$$K_3 = 2k_6k_7 - 2k_1k_3 \tag{6.32}$$

$$K_4 = -2k_2k_3 \tag{6.33}$$

$$K_5 = k_3^2 + k_5^2 - k_2^2 - k_7^2 \tag{6.34}$$

Solving equation 6.25 will generally yield four values of θ corresponding to the four possible limit positions. However, if the input link is a crank, no limit positions should exist, and, consequently, equation 6.25 should have no real roots. It is possible to find the roots of a quartic equation in closed form. However, to determine whether or not the input link is

a crank, it is only necessary to determine whether or not real roots of the quartic equation exist. Necessary and sufficient conditions for the absence of real roots in a quartic equation are given by Dickson (121). To apply these conditions, it is first necessary to replace x by z - B/4 in equation 6.25 to obtain the reduced quartic equation

$$z^4 + qz^2 + rz + s = 0 \qquad (6.35)$$

where

$$q = -3B^2/8 + C \qquad (6.36)$$

$$r = B^3/8 - CB/2 + D \qquad (6.37)$$

$$s = -3B^4/256 + CB^2/16 - DB/4 + E \qquad (6.38)$$

From the reduced quartic, the discriminant, Δ, and a quantity L are defined

$$\Delta = -4P^3 - 27Q^2 \qquad (6.39)$$

$$L = 8qs - 2q^3 - 9r^2 \qquad (6.40)$$

where

$$P = -4s - q^2/3 \qquad (6.41)$$

$$Q = 8qs/3 - r^2 - 2q^3/27 \qquad (6.42)$$

The absence of real roots is assured when the following conditions are satisfied (121, p. 100):

$$\Delta > 0 \tag{6.43}$$

$$q > 0 \qquad \text{or} \qquad L < 0 \tag{6.44}$$

Although somewhat complex to derive, the above expressions are simple, nonparametric constraint conditions which determine the input link rotatability of the RSSR mechanism. It should be noted that these same conditions may also be applied to the output link to determine its rotatability.

Again it should be emphasized that the RSSR-SC and RSSR-SS mechanisms may not have fully rotating input links even though the RSSR mechanisms may satisfy these conditions. On the other hand, these mechanisms may have fully rotating input links even though one of the RSSR mechanisms does not. In this case, the non-input revolute joint on the RSSR mechanism that does not have a fully rotating input link must be the joint replaced by a spheric or cylindric joint. Finally, if neither RSSR mechanism has a fully rotating input link, the input links of the RSSR-SC and RSSR-SS mechanisms will also not rotate fully.

6.5 The Transmission Characteristic Condition

As was the case for the Grashof condition in the previous section, the transmission characteristic

conditions developed in this section will not be exact for the RSSR-SC and RSSR-SS mechanisms. This is true because the transmission characteristics are of concern over a full cycle of the mechanism rather than at the precision positions alone. The approach taken will be to optimize the transmission characteristics of two RSSR mechanisms with one dyad in common. These mechanisms are than combined to give either the RSSR-SC or RSSR-SS mechanism. Once again, this approach will not guarantee an acceptable design. It does, however, provide a nonparametric basis for decision-making by which many unacceptable solutions may be rejected.

Following the work of Sutherland and Roth (122) and Söylemez and Freudenstein (78), the transmission ratio in the RSSR mechanism is defined as the fraction of force in the coupler which acts to produce output rotation. Using the notation of figure 6.6 and of the previous section, the transmission ratio, TR, may be expressed as

$$TR = [1 - \cos^2\mu - \sin^2\alpha]^{\frac{1}{2}} \tag{6.45}$$

where μ is the angle between the coupler link, b, and the output link, c, and α is the angle between the coupler link, b, and the plane normal to the unit vector along the output link axis, $\hat{\underset{\sim}{s}}_2$.

Using the geometric relations within the mechanism to replace μ and α in equation 6.45 leads to the result (122)

$$TR = \frac{1}{2bc}[4c^2(f\sin\psi - a\cos\psi\sin\theta)^2$$

$$+ 4c^2(e + a\cos\theta)^2$$

$$- (R + 2ae\cos\theta - 2ag\sin\psi\sin\theta)^2]^{\frac{1}{2}} \qquad (6.46)$$

where

$$R = a^2 - b^2 + c^2 + e^2 + f^2 + g^2 - 2fg\cos\psi \qquad (6.47)$$

The maximum and minimum values of TR may be determined by taking the derivative of the above expression with respect to θ, and equating the result with zero, to give

$$A\cos\theta + B\sin\theta + C\sin 2\theta + D\cos 2\theta = 0 \qquad (6.48)$$

where A, B, C and D are now defined by

$$A = Rg\sin\psi - 2c^2 f\sin\psi\cos\psi \qquad (6.49)$$

$$B = Re - 2c^2 e \qquad (6.50)$$

$$C = c^2 a\cos^2\psi + ae^2 - c^2 a - ag^2\sin^2\psi \qquad (6.51)$$

$$D = 2(aeg\sin\psi) \qquad (6.52)$$

At this point, Söylemez and Freudenstein (78) consider only special cases of the RSSR mechanism; these greatly simplify finding the roots of equation 6.48. To extend their procedure to the general case, it is necessary to find the roots of equation 6.48.

given any mechanism dimensions. This is done with the help of the following trigonometric identities:

$$\sin\theta = \frac{2t}{1 + t^2} \tag{6.53}$$

$$\cos\theta = \frac{1 - t^2}{1 + t^2} \tag{6.54}$$

$$\tan\theta = \frac{2t}{1 - t^2} \tag{6.55}$$

$$\sin 2\theta = \frac{2 \tan\theta}{1 + \tan^2\theta} = \frac{4t(1 - t^2)}{(1 - t^2)^2 + 4t^2} \tag{6.56}$$

$$\cos 2\theta = \frac{1 - \tan^2\theta}{1 + \tan^2\theta} = \frac{(1 - t^2)^2 - 4t^2}{(1 - t^2)^2 + 4t^2} \tag{6.57}$$

where $t = \tan \theta/2$. Substituting these into equation 6.47 yields, after some manipulation,

$$(D - A)t^4 + (2B - 4C)t^3 + (-6D)t^2$$

$$+ (2B + 4C)t + (A + D) = 0 \tag{6.58}$$

The real roots of the above quartic will correspond to the extreme values of the transmission ratio. The roots of equation 6.58 may be determined using any of the standard quartic root-finding procedures (121). For the purpose of working the example at the end of this chapter, optimum force transmission will be defined as the case when the minimum value of TR has been maximized.

Since two RSSR mechanisms are being considered simultaneously, the minimum value of TR must be selected between the two mechanisms.

6.6 Fixed-Pivot Location and Link-Length Ratio Conditions

Once again the constraint equations for these conditions will usually be self-evident and easy to apply. The fixed-pivot locations for the RSSR-SC and RSSR-SS mechanisms will be the same as for the RSSR-RS mechanism which in turn will be the same as for the pair of RSSR mechanisms with one dyad in common. A typical constraint condition may require the fixed pivots to be within a given radius of the origin of the global reference system. This could be expressed as

$$|\underset{\sim}{a}_0| \leq R_0 \tag{6.59}$$

$$|\underset{\sim}{b}_0| \leq R_0 \tag{6.60}$$

where R_0 is the given radius.

For the purposes of this section, the link lengths will be defined as the distances between consecutive joints in their starting position. Thus, for example, the ternary coupler link will result in three link lengths, and the entire mechanism will have a total of nine link lengths. As in the previous chapter, the ratio of the longest link length to the shortest link

length should not exceed a certain value, in this case ten

$$\frac{\text{length of longest link}}{\text{length of shortest link}} - 10 \leq 0 \tag{6.61}$$

6.7 Satisfying Additional Motion Requirements

Using the position stacking method presented in Chapter Five, it is possible to design the RSSR-SC and RSSR-SS mechanisms to satisfy up to four precision positions plus any additional number of approximate positions. The procedure is analogous to the procedure for the RC and CC dyads. The specified precision positions are divided into groups of three. The optimization process then seeks to find the most similar dyads satisfying each three-position motion group. The RS dyad constraint equations for position stacking must ensure similarity of the location, $\underset{\sim}{a}_0$, and the orientation, $\hat{\underset{\sim}{s}}_1$, of the revolute joint. Thus, for two groups of three precision positions, denoted by the superscripts 1 and 2, the function to be minimized is

$$F(\underset{\sim}{r}) = W_1|\underset{\sim}{a}_0^1 - \underset{\sim}{a}_0^2| + W_2|\hat{\underset{\sim}{s}}_1^1 - \hat{\underset{\sim}{s}}_1^2| \tag{6.62}$$

where $F(\underset{\sim}{r})$ shows $\underset{\sim}{a}_0$ and $\hat{\underset{\sim}{s}}_1$ to be functions of the location of the spheric joint, $\underset{\sim}{r}$, in the moving frame of reference and where W_1 and W_2 are weighting factors.

Of course, this procedure must be used in designing all three RS dyads and the same solution must be chosen in each case (i.e., either $\underset{\sim}{a}_0^1$, $\hat{\underset{\sim}{s}}_1^1$ or $\underset{\sim}{a}_0^2$, $\hat{\underset{\sim}{s}}_1^2$). The procedure just described will add some complexity to an already complex problem. Nevertheless, it is at least conceptually quite straightforward to apply. It also offers a solution to what seems to be a practical problem, namely, partially-exact partially-approximate synthesis. As shown in the next section, however, any attempt to satisfy four or more positions using the RS dyad must be done with careful consideration of the order problem.

6.8 The Order Condition

Section 5.5 discussed the general order problem in some detail. It was demonstrated that if the sequence of positions is correct, the order requirement can always be satisfied by choosing the correct direction of input link rotation. Furthermore, it was pointed out that the sequence of positions will usually be correct for a well-formulated problem.

This is extremely fortunate because, based on the findings of Sun and Waldron (123), it may be difficult or impossible to find RSSR mechanisms which satisfy a given set of rigid-body positions in more than one sequence. This conclusion was reached by noting that

points of the guided body which lie on a circle for four finitely separated spatial positions are distributed on a sixth-order spatial curve (110,111). Changes of sequence can only occur at points along this curve where the spheric joint has the same location for the two positions of the moving body experiencing a change in sequence (123). Since a body moving in space generally contains no points which remain stationary between two positions, sequence appears to be an invariable property of the design positions on a given branch of the sixth-order curve.

6.9 The Objective Function

As before, the objective function must include a term for each design requirement. Thus the objective function (O.F.) for the three precision position design of the RSSR-SC and RSSR-SS mechanisms may be expressed in symbolic form as

O.F. = B.A.T. + C.R.T. + T.C.T.

+ F.P.L.T. + L.L.R.T.

where

B.A.T. = Branch-Avoidance Term

C.R.T. = Crank-Rotatability Term

T.C.T. = Transmission Characteristic Term

F.P.L.T. = Fixed-Pivot Location Term

L.L.R.T. = Link-Length Ratio Term

The last two of these terms, F.P.L.T. and L.L.R.T., are handled in the same way as the similar conditions developed for the RCCC mechanism in Chapter Five. Additional motion requirements could be included in equation 6.63, as discussed in section 6.7. Notice that this would necessitate including an order of positions term in equation 6.63.

Transmission Characteristic Term. The third term of equation 6.63, the transmission characteristic term, can be taken to be the negative of the minimum value of the transmission ratio found in section 6.5, i.e.,

$$\text{T.C.T.} = -(TR_{\text{minimum}})$$

The negative sign is necessary because the best transmission corresponds to the largest value of TR. The first two terms of equation 6.63 are constraint conditions which must be handled using penalty function methods, as discussed below.

Branch-Avoidance Term. Equations 6.1 and 6.2 of section 6.3 give a nonparametric means for determining whether or not all the prescribed positions of the RSSR mechanism lie on one branch only. However, these conditions do not directly provide a measure of the amount by which a given mechanism fails to meet the branching condition. One such measure is now proposed.

First, define a new variable, BR_j, equal to the left-hand sides of equations 6.1 and 6.2 (which are the same),

$$BR_j = \overrightarrow{A_jB_j} \cdot \hat{\underset{\sim}{s}}_2 \times \overrightarrow{B_0B_j} \qquad (6.64)$$

Now if branching occurs, some of the BR_j will be greater than zero and some will be less than zero. In this case the objective should be to either force the negative values in the positive direction or force the positive values in the negative direction. The decision of which direction to move is based on which set has the largest sum magnitude. The branch-avoidance term thus becomes

$$B.A.T. = r\{\text{the lesser of } (\left|\sum_{j=1}^{n} \text{negative } BR_j\right|, \left|\sum_{j=1}^{n} \text{positive } BR_j)\right| \qquad (6.65)$$

where r is the penalty parameter. It can be seen from the above expression that when all the BR_j's are of the same sign, B.A.T. will be equal to zero.

Crank-Rotatability Term. Equations 6.43 and 6.44 give conditions which assure crank rotatability in the RSSR mechanism. Once again, however, it will be necessary to have some measure of the amount by which these conditions are violated. Using the logic symbols $\wedge$ and $\vee$ which respectively denote "and" and "or", the following equations satisfy the above-mentioned requirement:

$$\text{C.R.T.} = r\Delta \quad \text{if} \quad \Delta \leq 0 \wedge (q > 0 \vee L < 0) \tag{6.66}$$

$$\text{C.R.T.} = r\{\text{the lesser of } (|q|, |L|)\} \quad \text{if}$$

$$\Delta > 0 \wedge (q \leq 0 \vee L \geq 0) \tag{6.67}$$

$$\text{C.R.T.} = r\{\Delta + \text{the lesser of } (|q|, |L|)\} \quad \text{if}$$

$$\Delta \leq 0 \wedge (q \leq 0 \vee L \geq 0) \tag{6.68}$$

where, again, r is the penalty parameter.

The effect of these equations is to assess one penalty for violation of equation 6.43, and a second penalty for violation of equation 6.44. Since only one of the two expressions of equation 6.44 must be satisfied, the magnitude of the penalty is proportional to the lesser of the amounts by which the two terms are in violation.

A special problem arises in the application of the crank-rotatability conditions which merits additional discussion. When conditions 6.43 and 6.44 are satisfied, no real limit positions will exist for the input link of the RSSR mechanism. This situation occurs when either the input link of the mechanism is a crank, or when the mechanism cannot be assembled at all. The precision position synthesis procedure removes the latter possibility because it guarantees the mechanism can be assembled at the precision positions. However, this does prevent the solution from moving in this direction. In other words,

C.R.T., given by equations 6.66, 6.67 and 6.68, has a minimum value of zero when the input link is fully rotatable and when the mechanism cannot be assembled. Somewhere between these possibilities, it has a maximum value greater than zero. The optimization process may drive the solution toward the unassemblable minimum, even though it can never actually reach a value of zero. Because of this, it may be necessary to try several different starting points before an acceptable solution is found.

6.10 Numerical Example

The following positions and orientations of the moving body are specified:

$$\underset{\sim}{o}_1 = \{0\underset{\sim}{\hat{i}} + 0\underset{\sim}{\hat{j}} + 0\underset{\sim}{\hat{k}}\}^T$$

$$\underset{\sim}{o}_2 = \{1\underset{\sim}{\hat{i}} + 1\underset{\sim}{\hat{j}} + 1\underset{\sim}{\hat{k}}\}^T$$

$$\underset{\sim}{o}_3 = \{1\underset{\sim}{\hat{i}} + 2\underset{\sim}{\hat{j}} + 3\underset{\sim}{\hat{k}}\}^T$$

$$[R_2] = \begin{bmatrix} .868 & -.494 & .045 \\ .489 & .834 & -.255 \\ .089 & .243 & .966 \end{bmatrix}$$

$$[R_3] = \begin{bmatrix} .804 & -.590 & .071 \\ .577 & .745 & -.335 \\ .145 & .310 & .937 \end{bmatrix}$$

Again recall that $[R_1]$ is defined to be a 3x3 identity matrix without loss of generality.

Problem Statement. Design an RSSR-SR mechanism which satisfies the above precision positions while having acceptable transmission qualities. The solution mechanism is required to have a fully rotating input link and must be free from branching problems. Furthermore, the fixed pivots of the mechanism should be located within ten units of the global reference origin, and the ratio of the longest to the shortest link length should not exceed ten.

Solution. Solution mechanisms are obtained by minimizing the objective function of equation 6.63 using the Hooke and Jeeves optimization method. Appendix 2 provides a complete listing of all the APL-language computer programs used in solving this problem along with a sample run of the program package.

Figure 6.7 shows the RSSR-SR mechanism and associated design parameters in the initial position. Again it should be noted that, without loss of generality, the fixed and moving coordinate systems have been assumed to be coincident in the initial position. Tables 6.1 and 6.2 give the results of two separate program runs which converged to feasible solutions.

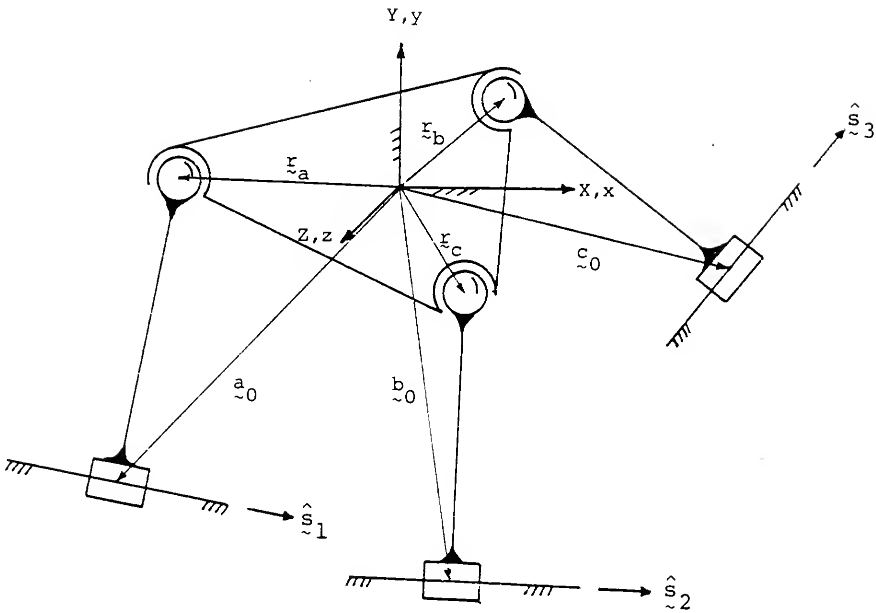

figure 6.7 The RSSR-SR mechanism and associated design parameters

Table 6.1

RSSR-SR Mechanism Optimization - Example 1

Assumed quantities (refer to figure 6.7)

$$\underset{\sim}{r}_a = \{8\hat{\underset{\sim}{i}} - 3\hat{\underset{\sim}{j}} + 3\hat{\underset{\sim}{k}}\}$$

$$\underset{\sim}{r}_b = \{7\hat{\underset{\sim}{i}} + 5\hat{\underset{\sim}{j}} - 2\hat{\underset{\sim}{k}}\}$$

$$\underset{\sim}{r}_c = \{8\hat{\underset{\sim}{i}} - 6\hat{\underset{\sim}{j}} + 3\hat{\underset{\sim}{k}}\}$$

After nine iterations of the APL program HOOKE and a total of 320 evaluations of the objective function, all the constraints were satisfied and the solution converged to the results given below.

	$\hat{\underset{\sim}{i}}$	$\hat{\underset{\sim}{j}}$	$\hat{\underset{\sim}{k}}$
$\underset{\sim}{r}_a = \underset{\sim}{a}_1 =$	8.355	-1.520	-1.400
$\underset{\sim}{r}_b = \underset{\sim}{b}_1 =$	6.200	2.080	.200
$\underset{\sim}{r}_c = \underset{\sim}{c}_1 =$	7.200	-6.538	-.200
$\underset{\sim}{a}_0 =$	6.546	.125	4.994
$\underset{\sim}{b}_0 =$	5.335	.761	6.605
$\underset{\sim}{c}_0 =$	6.850	-2.605	4.357
$\hat{\underset{\sim}{s}}_1 =$	.935	-.173	.309
$\hat{\underset{\sim}{s}}_2 =$	.978	.135	.160
$\hat{\underset{\sim}{s}}_3 =$	.771	-.451	.449

with a minimum value of the transmission ratio

$$TR = .663$$

Table 6.2

RSSR-SR Mechanism Optimization - Example 2

Assumed quantities (refer to figure 6.7)

$$\underset{\sim}{r}_a = \{3\hat{\underset{\sim}{i}} + 2\hat{\underset{\sim}{j}} - 2\hat{\underset{\sim}{k}}\}$$

$$\underset{\sim}{r}_b = \{5\hat{\underset{\sim}{i}} - 4\hat{\underset{\sim}{j}} + 2\hat{\underset{\sim}{k}}\}$$

$$\underset{\sim}{r}_c = \{6\hat{\underset{\sim}{i}} - 1\hat{\underset{\sim}{j}} - 4\hat{\underset{\sim}{k}}\}$$

After ten iterations of the APL program HOOKE and a total of 312 evaluations of the objective function, all the constraints were satisfied and the solution converged to the results given below.

	$\hat{\underset{\sim}{i}}$	$\hat{\underset{\sim}{j}}$	$\hat{\underset{\sim}{k}}$
$\underset{\sim}{r}_a = \underset{\sim}{a}_1 =$	3.940	2.925	-1.173
$\underset{\sim}{r}_b = \underset{\sim}{b}_1 =$	5.985	-3.924	-1.136
$\underset{\sim}{r}_c = \underset{\sim}{c}_1 =$	6.116	-1.000	-4.000
$\underset{\sim}{a}_0 =$	4.195	.394	5.340
$\underset{\sim}{b}_0 =$	5.143	-1.212	3.415
$\underset{\sim}{c}_0 =$	4.327	.243	2.286
$\hat{\underset{\sim}{s}}_1 =$	.952	.295	.077
$\hat{\underset{\sim}{s}}_2 =$	.824	-.406	.394
$\hat{\underset{\sim}{s}}_3 =$	.937	-.176	.301

with a minimum value of the transmission ratio

TR = .551

CHAPTER 7

CONCLUSIONS AND RECOMMENDATIONS FOR FUTURE RESEARCH

7.1 Conclusions

This dissertation has attempted to lay a groundwork for efficient mechanism design and optimization in the form of a general philosophy of mechanism optimization. The two most important points of this philosophy are (a) the need to express design requirements in the form of nonparametric constraint equations, and (b) the need to reduce the number of design parameters.

Design and optimization of mechanisms to meet all of the various real-world requirements have been emphasized, i.e., a new, "wholistic" approach was attempted. This is in contrast to many recent works dealing with mechanism optimization which consider only precision position synthesis and perhaps one or two additional design requirements. The role of precision position synthesis as a parameter reduction tool was discussed, along with the possibility of using other design requirements for parameter reduction. The formulation of nonparametric constraint equations representing the various design requirements was addressed in a number of places throughout the text. This process

was seen to be highly problem dependent, often requiring a great deal of insight into the special characteristics of a particular mechanism. The formulation of these nonparametric constraints is often the most critical step in the mechanism design process.

Development of the aforementioned philosophy was based, to a large extent, on a study of past research work and on a careful examination of current optimization theory. The optimization method of Hooke and Jeeves was selected for working the examples of this dissertation because, among other things, it is simple, requires minimal computer storage, can solve a large number of problem types and requires no derivatives of the objective function.

The philosophy developed in the first three chapters of this dissertation has been applied in Chapters Four, Five and Six. The types of mechanisms treated in the examples of these chapters are necessarily limited. Nevertheless, this work is believed to contain the most complete theories yet developed for the design of the RCCC, the RSSR-SC and the RSSR-SS spatial mechanisms. More importantly, perhaps, the examples serve to illustrate the previously-discussed philosophy, particularly the concept of parameter reduction and the formulation of nonparametric constraints.

Many of the theories developed in this dissertation

can be used directly to solve related problems. For example, in Chapter Six, constraint conditions were developed for the RSSR mechanism as an intermediate step in the design of the RSSR-SC and the RSSR-SS mechanisms. These constraint conditions are, however, directly applicable to the RSSR function-generating mechanism. Also, the branching condition for the spherical RRRR mechanism developed in Chapter Five applies directly to that mechanism as well as to the equivalent RCCC spatial mechanism.

The use of the position stacking method for the partially-exact partially-approximate synthesis of spatial mechanisms has been developed and applied. This novel approach allows the designer to satisfy up to the maximum number of precision positions and, at the same time, approximate any number of additional positions. One of the primary benefits of this method is the parameter reduction which accompanies precision position synthesis.

Many of the penalty function expressions developed in Chapters Five and Six are believed to be unique to this work. An example of this is the branch-avoidance term for the RCCC mechanism, given in equation 5.24. This expression serves as a measure of the amount by which the branching constraint is violated.

7.2 Recommendations for Future Research

It is believed that the material in this dissertation represents a significant step forward in the theory and

the practice of mechanism optimization. Yet it must be recognized that all the research to date in this area represents only a small fraction of what can potentially be accomplished. This section should by no means be considered a complete listing of the unsolved mechanism optimization problems. Rather, it is a brief summary of a few of the more obvious problems which would follow as an extension and an improvement of this work.

One of the most important and most difficult topics which, to a large extent, remains to be studied, is mechanism dynamics. This area includes balancing and kineto-elasto-dynamics as well as force and torque analysis. Improved theories are needed for both planar and spatial mechanisms which will allow the nonparametric prediction of critical dynamic properties. At present, dynamic analysis generally employs a position, velocity and acceleration analysis for closely-spaced values of the input parameter. This type of parametric approach is very inefficient in an optimization loop.

Another important consideration in the design of mechanisms is the effect of tolerance and clearance on the specified performance. No precision position can truly be precise, and no mechanism dimension can be totally accurate. Obviously, the effect of these on mechanism performance must be held within a certain range if the mechanism is to be acceptable. But what is this

range and how can it be determined nonparametrically? Notice that when tolerance or clearance is present, all of the design requirements contain some degree of uncertainty. This points towards the exploration of a statistical or stochastic approach.

Theories for the design and optimization of spatial mechanisms are in their infancy. It is not yet clear which spatial mechanisms are useful and for what purposes. The questions about the transmission characteristics of the RCCC mechanism leave some doubt about its practical usefulness. Although single-degree-of-freedom seven-link spatial mechanisms are well known (116), design procedures are available for only the simplest three-, four- and five-link mechanisms. Even these theories are quite limited in their scope of application, and often only apply to special cases of these mechanisms.

To date, very little consideration has been given to mechanism workspace requirements. Since these mechanisms will inevitably be part of a machine, interference with surrounding components must be avoided. This is likely to be a much greater problem in spatial mechanism design than in planar mechanism design since the motions are usually much more difficult to visualize and to represent on graphic computer displays.

The author realizes that many of the problems that have been so casually mentioned here will be extremely

difficult to solve. And, of course, countless other important problems exist which have not been mentioned here. It is doubtful that the goal of truly complete mechanism design theories will be attained in the foreseeable future. There will always be higher-order properties and unique situations to challenge the ingenuity of the designer. Still, the demands of competitive industry and of an increasingly mechanized world make improved design theories an urgent necessity. The author hopes this dissertation has made at least a small contribution toward the goal of improved mechanism design theories.

APPENDIX 1

RCCC MECHANISM OPTIMIZATION PROGRAMS AND SAMPLE RUN

This appendix contains a complete listing of the APL-language computer programs required for optimizing the RCCC mechanism as discussed in Chapter Five. A sample computer output of a typical run of the program package follows the program listings. A brief description of each program is given below.

HOOKE	This program seeks the minimum value of the function FUN given an arbitrary starting point. It is based on the Hooke and Jeeve's optimization procedure described in section 2.4.4.
FUN	This program calculates the objective function as given by equation 5.20.
UNITVEC	This program calculates a unit vector given two angles (in degrees). These two angles serve as the design variables.
<u>MET</u>	This program calculates the motion error term (M.E.T.) of equation 5.20.
RCCCSYN	This program is called by <u>MET</u>. It calculates the dimensions of an RCCC mechanism given the two fixed-axis directions and a pair of scalar displacements (see Chapter Four).
CCDYAD	This program is called by RCCCSYN. It calculates the dimensions of an RC or CC dyad for three precision positions of rigid-body guidance.
AXIS	This program is called by CCDYAD. It calculates the moving dyad joint axis direction given the fixed axis direction.

CROSS	This program calculates the cross-product of two vectors.
CRT	This program calculates the crank-rotatability term (C.R.T.) of equation 5.20.
BAT	This program calculates the branch-avoidance term (B.A.T.) of equation 5.20.
OPT	This program calculates the order-of-positions term (O.P.T.) of equation 5.20.
FPLT	This program calculates the fixed-pivot location term (F.P.L.T.) of equation 5.20.
LLRT	This program calculates the link-length ratio term (L.L.R.T.) of equation 5.20.
FBCALC	This program is called by BAT. It performs the repetitive calculation of equation 5.23.
EULER	This program is not directly used in the optimization package. It calculates the rotation matrix given a set of three Euler angles (see section 4.3).
PRINT	This program is called by HOOKE. It prints output information at prespecified intervals (see the sample program run which follows).

RCCC Mechanism Optimization Programs

```
    ∇ HOOKE
[1]    'INPUT STARTING POINT'
[2]    N←ρX0←⎕
[3]    'INPUT INITIAL STEP SIZE VECTOR'
[4]    A←⎕
[5]    'INPUT MINIMUM STEP SIZE'
[6]    AMIN←⎕
[7]    'INPUT PENALTY PARAMETER VALUE'
[8]    PEN←⎕
[9]    'INPUT MAXIMUM NUMBER OF ITERATIONS'
[10]   MMAX←⎕
[11]   'INPUT THE NUMBER OF ITERATIONS BETWEEN PRINTINGS'
[12]   MPRINT←⎕
[13]   ΔX←10*¯6
[14]   M←0
[15]   FE←0
[16]  L4:X←X0
[17]  L6:Q←1,0↑M←M+1
[18]   →(L8,L13,L14,L14)[1+(0=MPRINT|M)+2×MMAX=M]
[19]  L13:PRINT X,A
[20]  L8:X1←X0
[21]   X1[Q]←X1[Q]+A[Q]
[22]   →(L2,L1)[1+(FUN X1)<FUN X0]
[23]  L1:X0←X1
[24]   A[Q]←1.2×A[Q]
[25]   →L7
[26]  L2:X1[Q]←X1[Q]+2×A[Q]←-A[Q]
[27]   →(L3,L1)[1+(FUN X1)<FUN X0]
[28]  L3:A[Q]←A[Q]÷2
[29]  L7:→(L8,L5)[1+N<Q←Q+1]
[30]  L5:→(L9,L11)[1+∧/(|X0-X)<ΔX]
[31]  L9:X1←(2×X0)-X
[32]   →(L4,L10)[1+(FUN X1)<FUN X0]
[33]  L10:X←X0←X1
[34]   →L6
[35]  L11:→(L6,L12)[1+∧/(|A)<AMIN]
[36]  L14:'*****ITERATION LIMIT REACHED*****'
[37]   →L15
[38]  L12:'*****SOLUTION HAS CONVERGED*****'
[39]  L15:PRINT X0,A
    ∇
```

Programs (continued)

```
     ∇ OF←FUN SD
[1]    FE←FE+1
[2]    S1←UNITVEC 2↑SD
[3]    S4←UNITVEC 2↑2⌽SD
[4]    SS←S1,S4,¯2↑SD
[5]    MET←MET SS
[6]    AXES←(6↑V1),(3↑17⌽V1),3↑14⌽V1
[7]    CRT←CRT AXES
[8]    BAT←BAT AXES
[9]    OPT←OPT(6↑AXES),¯3↑AXES
[10]   FPLT←FPLT(3↑9⌽V1),3↑23⌽V1
[11]   LLRT←LLRT(3↑9⌽V1),(○1+3↑6⌽V1),(○1+3↑20⌽V1),3↑23⌽V1
[12]   OF←MET+PEN×(100×CRT)+BAT+OPT+(1↑FPLT)+1↑LLRT
[13]   OF←OF+FPLT[2]+LLRT[2]
     ∇
```

```
     ∇ U0←UNITVEC A;U0X
[1]    A←(○1÷180)×A
[2]    U0X←(1÷1+((3○A[1])*2)+(3○A[2])*2)*.5
[3]    U0←U0X,(U0X×3○A[1]),U0X×3○A[2]
     ∇
```

```
     ∇ MET←MET SS
[1]    R13←RA
[2]    V1←RCCCSYN SS
[3]    R13←RB
[4]    V2←RCCCSYN SS
[5]    V1P←(3↑9⌽V1),(3↑3⌽V1),(3↑6⌽V1)
[6]    V1P←V1P,(3↑23⌽V1),(3↑17⌽V1),3↑20⌽V1
[7]    V2P←(3↑9⌽V2),(3↑3⌽V2),3↑6⌽V2
[8]    V2P←V2P,(3↑23⌽V2),(3↑17⌽V2),3↑20⌽V2
[9]    MET←+/((+/(6 3⍴V1P-V2P)*2)*.5)×1 100 1 1 100 1
     ∇
```

```
     ∇ V←RCCCSYN SS;S1RC;S12;S13;S1CC
[1]    S1RC←AXIS 3↑SS
[2]    S12←0⋄ S13←0
[4]    V←CCDYAD(3↑SS),0,0,S1RC
[5]    V←(3↑SS),S1RC,V
[6]    S1CC←AXIS 3↑3⌽SS
[7]    S12←1↑6⌽SS⋄ S13←1↑7⌽SS
[9]    V←V,(3↑3⌽SS),S1CC,CCDYAD(3↑3⌽SS),S12,S13,S1CC
     ∇
```

```
     ∇ RAS←CCDYAD SS;U2;U1;U3;M;U0;C;UC1;UC2;UC3
[1]    U0←3↑SS⋄ U1←¯3↑SS⋄ S12←SS[4]⋄ S13←SS[5]
[5]    U2←R12+.×U1⋄ U3←R13+.×U1
[7]    UC1←U1 CROSS U0⋄ UC2←U2 CROSS U0⋄ UC3←U3 CROSS U0
[10]   M←U0,(-U0),0,0,((⍉R12)+.×U0),(-U0),(-+/U0×U2),0
[11]   M←M,((⍉R13)+.×U0),(-U0),0,(-+/U0×U3),U1,(-U1),0,0
[12]   M←M,U1,(-U2),¯1,0,U1,(-U3),0,¯1
[13]   M←M,(((⍉R12)+.×UC2)-UC1),(UC1-UC2),0,0
[14]   M←M,(((⍉R13)+.×UC3)-UC1),(UC1-UC3),0,0
[15]   M←8 8⍴M
[16]   C←(-+/U0×O1),(S12-+/U0×O2),(S13-+/U0×O3)
[17]   C←C,(-+/U1×O1),((+/U2×S12×U0)-+/U2×O2)
[18]   C←C,((+/U3×S13×U0)-+/U3×O3)
[19]   C←C,((-+/U0×O2 CROSS U2)++/U0×O1 CROSS U1)
[20]   C←C,((-+/U0×O3 CROSS U3)++/U0×O1 CROSS U1)
[21]   RAS←C⌹M
     ∇


     ∇ U1←AXIS U0;A;B;C;D;U1Z
[1]    ⍝GIVEN THE 1ST AXIS OF AN R-C OR C-C DYAD
[2]    ⍝ AND 3 PRESCRIBED POSITIONS, CALC THE 2ND AXIS
[3]    ⍝ROTATIONS R12 AND R13 MUST BE GLOBAL VARS
[4]    A←((⍉R12)-(⍳3)∘.=⍳3)+.×U0
[5]    B←((⍉R13)-(⍳3)∘.=⍳3)+.×U0
[6]    C←((B[2]×A[3])-B[3]×A[2])÷(B[1]×A[2])-B[2]×A[1]
[7]    D←((B[1]×A[3])-B[3]×A[1])÷(B[2]×A[1])-B[1]×A[2]
[8]    U1Z←(1÷(C*2)+(D*2)+1)*.5
[9]    U1←(C×U1Z),(D×U1Z),U1Z
     ∇


     ∇ C←A CROSS B
[1]    C←1⊖(A×1⌽B)-B×1⊖A
     ∇


     ∇ CRT←CRT U
[1]    ALPHA←¯2○+/U×1⌽[1]U←4 3⍴U
[2]    ALPHA←(ALPHA×ALPHA≤○.5)+((○1)-ALPHA)×ALPHA>○.5
[3]    ΔALPHA←(1↑ALPHA)-1↑ALPHA[⍋ALPHA]
[4]    ΔALPHA←10×ΔALPHA+ΔALPHA>0
[5]    CRT←.01+((○.5)+1↑ALPHA)-.5×+/ALPHA
[6]    CRT←(CRT×CRT>0)+ΔALPHA
[7]    CRT←CRT+CRT>0
     ∇
```

```
     ∇ BAT←BAT U;FB;FBG;FBL
[1]    S←U←4 3⍴U
[2]    FB←FBCALC U
[3]    S[2;]←R12+.×U[2;]⋄ S[3;]←R12+.×U[3;]
[5]    FB←FB,FBCALC S
[6]    S[2;]←RA+.×U[2;]⋄ S[3;]←RA+.×U[3;]
[8]    FB←FB,FBCALC S
[9]    S[2;]←RB+.×U[2;]⋄ S[3;]←RB+.×U[3;]
[11]   FB←FB,FBCALC S
[12]   BAT←(+/FB×FB>.01)⌊|+/FB×FB<.01
[13]   BAT←BAT+BAT≠0
     ∇
```

```
     ∇ OPT←OPT S;A;A41;TH;ORD;IF;B
[1]    S1←3↑S⋄ S21←3↑3⌽S⋄ S41←¯3↑S
[4]    A←(S1 CROSS S21),S1 CROSS R12+.×S21
[5]    A←A,(S1 CROSS RA+.×S21),S1 CROSS RB+.×S21
[6]    A←4 3⍴A
[7]    A41←S41 CROSS S1
[8]    IF←A+.×A41 CROSS S1
[9]    TH←((¯2○A+.×A41)×IF≥0)+((○2)-¯2○A+.×A41)×IF<0
[10]   ORD←⍋TH
[11]   B←1 2 3 4 1 4 3 2 1 3 2 4 1 4 2 3
[12]   B←6 4⍴B,1 2 4 3 1 3 4 2
[13]   T←(+/(6 4⍴(¯1+ORD⍳1)⌽ORD)=B)⍳4
[14]   →(L1,L1,L2,L2,L3,L3)[T]
[15] L1:OPT←0
[16]   →LA
[17] L2:OPT←(|(TH[ORD⍳2])-TH[ORD⍳3])
[18]   OPT←OPT⌊|TH[ORD⍳1]-TH[ORD⍳4]
[19]   →LA
[20] L3:OPT←(|TH[ORD⍳1]-TH[ORD⍳2])
[21]   OPT←OPT⌊|(TH[ORD⍳3])-TH[ORD⍳4]
[22] LA:OPT←OPT+OPT>0
     ∇
```

```
     ∇ FPLT←FPLT AB;T
[1]    T←((+/(2 3⍴AB)*2)*.5)-100
[2]    FPLT←(+/T×T>0),+/T+100
     ∇
```

```
     ∇ LLRT←LLRT JL;LL
[1]    LL←LL[⍋LL←+/4 3⍴|JL-3⌽JL]
[2]    LL←LL[4]÷LL[1]
[3]    LLRT←((LL>10)×LL),LL
     ∇
```

Programs (continued)

```
    ∇ T←FBCALC S
[1]   T←((S[3;]-S[2;])CROSS S[4;]-S[3;])+.×S[3;]
    ∇
```

```
    ∇ RJ←EULER ANG;C;S
[1]   ANG←ANG×○÷180
[2]   C←2○ANG
[3]   S←1○ANG
[4]   RJ←((C[1]×C[3])-S[1]×S[3]×C[2])
[5]   RJ←RJ,(¯1×(C[1]×S[3])+S[1]×C[3]×C[2]),S[1]×S[2]
[6]   RJ←RJ,((S[1]×C[3])+C[1]×C[2]×S[3])
[7]   RJ←RJ,((¯1×S[1]×S[3])+C[1]×C[2]×C[3]),(¯1×C[1]×S[2])
[8]   RJ←(3 3)⍴RJ,(S[2]×S[3]),(S[2]×C[3]),C[2]
    ∇
```

```
    ∇ PRINT X
[1]   ''
[2]   'ITERATION NUMBER ';M
[3]   'CURRENT DESIGN VECTOR ';6↑X
[4]   'CURRENT STEP VECTOR ';¯6↑X
[5]   'OBJECTIVE FUNCTION VALUE ';FUN 6↑X
[6]   'CONSTRAINT CONDITION VALUES'
[7]   'CRT=';CRT
[8]   'BAT=';BAT
[9]   'OPT=';OPT
[10]  'FPLT=';1↑FPLT
[11]  'LLRT=';1↑LLRT
[12]  'TOTAL NUMBER OF FUNCTIONAL EVALUATIONS=';FE
[13]  ''
    ∇
```

RCCC Mechanism Optimization Sample Run

```
        O1
0 0 0
        O2
4 2 0
        O3
8 2 3
        O4
10 3 5
        R12
 .96598 ¯.25849 _.00759
 .25816  .96219 ¯.08682
 .01513  .08583  .99619
        RA
 .86805 ¯.49444 _.04494
 .48852  .83449 ¯.25489
 .08852  .24321  .96593
        RB
 .82358 ¯.56409 _.05939
 .54848  .76533 ¯.33682
 .14454  .30998  .93969


       HOOKE
INPUT STARTING POINT
⎕:
       270 45 190 175 3 3
INPUT INITIAL STEP SIZE VECTOR
⎕:
       25 25 25 25 3 3
INPUT MINIMUM STEP SIZE
⎕:
       .01
INPUT PENALTY PARAMETER VALUE
⎕:
       1000
INPUT MAXIMUM NUMBER OF ITERATIONS
⎕:
       25
INPUT THE NUMBER OF ITERATIONS BETWEEN PRINTINGS
⎕:
       5
```

Sample Run (continued)

```
ITERATION NUMBER 1
CURRENT DESIGN VECTOR 270 45 190 175 3 3
CURRENT STEP VECTOR 25 25 25 25 3 3
OBJECTIVE FUNCTION VALUE 2.5769E6
CONSTRAINT CONDITION VALUES
CRT=11.923
BAT=0
OPT=0
FPLT=1381.8
LLRT=0
TOTAL NUMBER OF FUNCTIONAL EVALUATIONS=1


ITERATION NUMBER 5
CURRENT DESIGN VECTOR 302.5 62.5 202.5 200 0 .6
CURRENT STEP VECTOR 9 ¯9 3.75 ¯3.75 .45 ¯1.08
OBJECTIVE FUNCTION VALUE 148.01
CONSTRAINT CONDITION VALUES
CRT=0
BAT=0
OPT=0
FPLT=0
LLRT=0
TOTAL NUMBER OF FUNCTIONAL EVALUATIONS=96


ITERATION NUMBER 10
CURRENT DESIGN VECTOR 304.75 63.625 202.03 200.47
      .02812 .6
CURRENT STEP VECTOR .675 ¯.675 .28125 ¯.28125 .03375
      .03375
OBJECTIVE FUNCTION VALUE 132.37
CONSTRAINT CONDITION VALUES
CRT=0
BAT=0
OPT=0
FPLT=0
LLRT=0
TOTAL NUMBER OF FUNCTIONAL EVALUATIONS=213
```

Sample Run (continued)

```
ITERATION NUMBER 15
CURRENT DESIGN VECTOR 303.91 66.506 201.58 201.98
      ¯.05017 .74404
CURRENT STEP VECTOR .2916 .69984 ¯.05062 .2916 ¯.01458
      .03499
OBJECTIVE FUNCTION VALUE 113.67
CONSTRAINT CONDITION VALUES
CRT=0
BAT=0
OPT=0
FPLT=0
LLRT=0
TOTAL NUMBER OF FUNCTIONAL EVALUATIONS=319


ITERATION NUMBER 20
CURRENT DESIGN VECTOR 300.28 68.13 201.25 202.36
      ¯.06912 .74842
CURRENT STEP VECTOR ¯.72559 .30233 ¯.05248 .05248
      ¯.00262 .00262
OBJECTIVE FUNCTION VALUE 107.14
CONSTRAINT CONDITION VALUES
CRT=0
BAT=0
OPT=0
FPLT=0
LLRT=0
TOTAL NUMBER OF FUNCTIONAL EVALUATIONS=428

*****ITERATION LIMIT REACHED*****

ITERATION NUMBER 25
CURRENT DESIGN VECTOR 300.01 68.111 201.19 202.55
      ¯.08094 .7649
CURRENT STEP VECTOR .7523 ¯.02267 .00944 ¯.02267
      ¯.00272 .00113
OBJECTIVE FUNCTION VALUE 105.3
CONSTRAINT CONDITION VALUES
CRT=0
BAT=0
OPT=0
FPLT=0
LLRT=0
TOTAL NUMBER OF FUNCTIONAL EVALUATIONS=539
```

APPENDIX 2

RSSR-SR MECHANISM OPTIMIZATION PROGRAMS AND SAMPLE RUN

This appendix contains a complete listing of the APL-language computer programs required for optimizing the RSSR-SR mechanism as discussed in Chapter Six. A sample computer output of a typical run of the program package follows the program listings. A brief description of each program is given below. It should be noted that only the HOOKE program is the same as was used in the RCCC mechanism optimization. Other programs with the same name, for example CRT, serve the same purpose as before; however, the programs are not the same.

HOOKE	This program seeks the minimum value of the function FUN given an arbitrary starting point. It is based on the Hooke and Jeeve's optimization procedure described in section 2.4.4.
FUN	This program calculates the objective function as given by equation 6.63.
RSSRSRSYN	This program calculates the fixed-pivot locations and axis orientations of the RSSR-SR mechanism given the locations of the three spheric joints.
BAT	This program calculates the branch-avoidance term (B.A.T.) of equation 6.63.
MECHDIM	This program calculates the dimensions of the RSSR-SR mechanism in the coordinate system of figure 6.6.

CRT	This program calculates the crank-rotatability term (C.R.T.) of equation 6.63.
LLRT	This program calculates the link-length ratio term (L.L.R.T.) of equation 6.63.
TR	This program calculates the transmission characteristic term (T.C.T.) of equation 6.63.
FPLT	This program calculates the fixed-pivot location term of equation 6.63.
RSDYAD	This program is called by RSSRSRSYN. It calculates the fixed pivot location and axis orientation of an RS dyad given the initial spheric joint location.
REALROOT	This program is called by CRT. It determines whether or not a given quartic equation has real roots.
CROSS	This program calculates the cross-product of two vectors.
PRINT	This program is called by HOOKE. It prints output information at prespecified intervals (see the sample program run which follows).
EULER	This program is not directly used in the optimization package. It calculates the rotation matrix given a set of three Euler angles (see section 4.3).

The following programs were authored by Dr. Gary K. Matthew. They calculate the real and imaginary roots of a quartic equation and are called by TR.

QUART
CUBIC
QUAD
CROOT
CR
ATAN

RSSR-SR Mechanism Optimization Programs

```
     ∇ HOOKE
[1]    'INPUT STARTING POINT'
[2]    N←ρX0←⎕
[3]    'INPUT INITIAL STEP SIZE VECTOR'
[4]    A←⎕
[5]    'INPUT MINIMUM STEP SIZE'
[6]    AMIN←⎕
[7]    'INPUT PENALTY PARAMETER VALUE'
[8]    PEN←⎕
[9]    'INPUT MAXIMUM NUMBER OF ITERATIONS'
[10]   MMAX←⎕
[11]   'INPUT THE NUMBER OF ITERATIONS BETWEEN PRINTINGS'
[12]   MPRINT←⎕
[13]   ΔX←10*¯6
[14]   M←0
[15]   FE←0
[16] L4:X←X0
[17] L6:Q←1,0↑M←M+1
[18]   →(L8,L13,L14,L14)[1+(0=MPRINT|M)+2×MMAX=M]
[19] L13:PRINT X,A
[20] L8:X1←X0
[21]   X1[Q]←X1[Q]+A[Q]
[22]   →(L2,L1)[1+(FUN X1)<FUN X0]
[23] L1:X0←X1
[24]   A[Q]←1.2×A[Q]
[25]   →L7
[26] L2:X1[Q]←X1[Q]+2×A[Q]←-A[Q]
[27]   →(L3,L1)[1+(FUN X1)<FUN X0]
[28] L3:A[Q]←A[Q]÷2
[29] L7:→(L8,L5)[1+N<Q←Q+1]
[30] L5:→(L9,L11)[1+∧/(|X0-X)<ΔX]
[31] L9:X1←(2×X0)-X
[32]   →(L4,L10)[1+(FUN X1)<FUN X0]
[33] L10:X←X0←X1
[34]   →L6
[35] L11:→(L6,L12)[1+∧/(|A)<AMIN]
[36] L14:'*****ITERATION LIMIT REACHED*****'
[37]   →L15
[38] L12:'*****SOLUTION HAS CONVERGED*****'
[39] L15:PRINT X0,A
     ∇
```

Programs (continued)

```
      ∇ OF←FUN RRR
 [1]    FE←FE+1
 [2]    SA←RSSRSRSYN RRR
 [3]    BAT1←BAT(3↑3⌽RRR),3↓18⌽SA
 [4]    DIM1←MECHDIM 18↑SA
 [5]    CRT1←CRT DIM1
 [6]    TR1←TR DIM1
 [7]    LLRT1←LLRT DIM1
 [8]    BAT2←BAT(3↑6⌽RRR),(3↓9↑SA),¯9↑SA
 [9]    DIM2←MECHDIM(9↑SA),¯9↑SA
 [10]   CRT2←CRT DIM2
 [11]   TR2←TR DIM2
 [12]   LLRT2←LLRT DIM2
 [13]   FPLT←FPLT SA
 [14]   OF←BAT1+BAT2+10×CRT1+CRT2
 [15]   OF←(-1↑(TR1,TR2)[⍋TR1,TR2])+OF
 [16]   OF←OF+LLRT1+LLRT2+FPLT
      ∇
```

```
      ∇ SA←RSSRSRSYN RRR
 [1]    SA←RSDYAD 3↑RRR
 [2]    SA←SA,RSDYAD 3↑3⌽RRR
 [3]    SA←SA,RSDYAD ¯3↑RRR
      ∇
```

```
      ∇ BAT←BAT X;S2;RB;AB;B0;C;T1;T2
 [1]    S2←3↑9⌽X◇ RB←3↑X
 [3]    AB←(¯3↑X)-3↑6⌽X◇ B0←3↑12⌽X
 [5]    C←AB+.×S2 CROSS(O[1;]+RB)-B0
 [6]    C←C,(R2+.×AB)+.×S2 CROSS(O[2;]+R2+.×RB)-B0
 [7]    C←C,(R3+.×AB)+.×S2 CROSS(O[3;]+R3+.×RB)-B0
 [8]    T1←+/C×C≥0
 [9]    T2←|+/C×C≤0
 [10]   →(LA,LB)[1+(T1=0)∨T2=0]
 [11] LA:BAT←T1⌊T2
 [12]   BAT←BAT+BAT>0
 [13]   →0
 [14] LB:BAT←0
      ∇
```

Programs (continued)

```
     ∇ DIM←MECHDIM A1A2;A;B;C;E;F;G;PSI;S1;S2;A0;A1;B0;B1;
       T1;T2
[1]    S1←3↑A1A2◇ A0←3↑3⌽A1A2◇ A1←3↑6⌽A1A2
[4]    S2←3↑9⌽A1A2◇ B0←3↑12⌽A1A2◇ B1←3↑15⌽A1A2
[7]    A←((A1-A0)+.×A1-A0)*.5
[8]    C←((B1-B0)+.×B1-B0)*.5
[9]    B←((A1-B1)+.×A1-B1)*.5
[10]   T1←S1 CROSS S2
[11]   E←|(A0-B0)+.×T1÷(+/T1*2)*.5
[12]   F←((A0-B0)+.×S2 CROSS T1)÷T1+.×T1
[13]   G←-((A0-B0)+.×S1 CROSS-T1)÷T1+.×T1
[14]   PSI←¯2○S1+.×S2
[15]   PSID←PSI×180÷○1
[16]   DIM←A,B,C,E,F,G,PSI
     ∇


     ∇ CRT←CRT D;T1;SK1;SK2;SK3;SK4;SK5;SK6;K1;K2;K3;K4;K5
       ;B1;C1;D1;E1;SQ;SR;SS;A;B;C;E;F;G;PSI
[1]    A←D[1]◇ B←D[2]◇ C←D[3]◇ E←D[4]
[5]    F←D[5]◇ G←D[6]◇ PSI←D[7]
[8]    SK1←(F*2)+(A*2)+(E*2)+(G*2)+(C*2)-B*2
[9]    SK1←(F×G×(2○PSI))-.5×SK1
[10]   SK2←A×E◇ SK3←A×G×1○PSI
[12]   SK4←-C×E◇ SK5←C×A
[14]   SK6←C×F×1○PSI◇ SK7←-C×A×2○PSI
[16]   K1←(SK4*2)+(SK6*2)+(SK7*2)-(SK1*2)+SK3*2
[17]   K2←(2×SK4×SK5)-2×SK1×SK2
[18]   K3←(2×SK6×SK7)-2×SK1×SK3
[19]   K4←¯2×SK2×SK3
[20]   K5←(SK3*2)+(SK5*2)-(SK2*2)+SK7*2
[21]   B1←((2×K2×K5)+2×K3×K4)÷T1←(K4*2)+K5*2
[22]   C1←((K2*2)+(K3*2)+(2×K1×K5)-K4*2)÷T1
[23]   D1←((2×K1×K2)-2×K3×K4)÷T1
[24]   E1←((K1*2)-K3*2)÷T1
[25]   CRT←REALROOT 1,B1,C1,D1,E1
[26]   CRT←CRT+CRT>0
[27]   TEMP←1,B1,C1,D1,E1
     ∇


     ∇ LLRT←LLRT DIM
[1]    LLRT←(4↑DIM)[⍋4↑DIM]
[2]    LLRT←LLRT[4]÷LLRT[1]
[3]    LLRT←LLRT×LLRT>10
     ∇
```

Programs (continued)

```
     ∇ TI←TR D;A;B;C;E;F;G;PSI;CA;CB;CC;TR1;TR2;TR3
[1]    A←D[1]◇ B←D[2]◇ C←D[3]◇ E←D[4]
[5]    F←D[5]◇ G←D[6]◇ PSI←D[7]
[8]    R←(A*2)+(C*2)+(E*2)+(F*2)+G*2
[9]    R←R-(B*2)+2×F×G×2○PSI
[10]   CA←(1○PSI)×(R×G)-2×(C*2)×F×2○PSI
[11]   CB←E×R-2×C*2
[12]   CC←(2×(C*2)×(2○PSI)*2)+2×E*2
[13]   CC←.5×A×CC-(2×(C*2))+2×(G*2)×(1○PSI)*2
[14]   CD←2×A×E×G×1○PSI
[15]   T←(CD-CA),(2×CB-2×CC),(¯6×CD),(2×CB+2×CC),CA+CD
[16]   TH←QUART T
[17]   TH←(,1⌽TH=1ρ0)/,TH
[18]   TH←2×¯3○TH
[19]   TR1←(R+(2×A×E×2○TH)-2×A×G×(1○PSI)×(1○TH))*2
[20]   TR2←4×(C*2)×(E+A×2○TH)*2
[21]   TR3←4×(C*2)×((F×1○PSI)-A×(2○PSI)×1○TH)*2
[22]   TR←(TR3+TR2-TR1)÷4×(B*2)×C*2
[23]   TI←((TR>0)/TR)*.5
     ∇
```

```
     ∇ FPLT←FPLT SA
[1]    A0←3↑3⌽SA◇ B0←3↑12⌽SA◇ C0←3↑21⌽SA
[4]    FP←((+/A0*2),(+/B0*2),+/C0*2)*.5
[5]    FPLT←+/FP×FP>5
     ∇
```

```
     ∇ Z←RSDYAD X;A;P2;P3;C1;C2;S;A0;L3;RJ
[1]    R←X
[2]    RJ←(3 3)ρ(R1+.×R),(R2+.×R),R3+.×R
[3]    A←0J+RJ
[4]    S←(A[2;]-A[1;])CROSS A[3;]-A[1;]
[5]    S←S÷(+/S*2)*.5
[6]    P2←S CROSS P2÷(+/(P2←A[2;]-A[1;])*2)*.5
[7]    P3←S CROSS P3÷(+/(P3←A[3;]-A[2;])*2)*.5
[8]    C1←(P2 CROSS A[1;])+P2 CROSS(A[2;]-A[1;])÷2
[9]    C2←(P2 CROSS A[2;])+P2 CROSS(A[3;]-A[2;])÷2
[10]   L3←(L3+.×C1-C2)÷L3+.×L3←P2 CROSS P3
[11]   A0←(L3×P3)+(A[3;]+A[2;])÷2
[12]   Z←S,A0,A[1;]
     ∇
```

Programs (continued)

```
      ∇ Δ←REALROOT X;Q;R;S;P
[1]     Q←((¯3×X[2]*2)÷8)+X[3]
[2]     R←X[4]+((X[2]*3)÷8)-(X[2]×X[3])÷2
[3]     S←X[5]+(-(X[4]×X[2])÷4)+((X[3]×X[2]*2)÷16)
[4]     S←S-(3×X[2]*4)÷256
[5]     P←-(4×S)+(Q*2)÷3
[6]     CQ←((8×Q×S)÷3)-(R*2)+(2×Q*3)÷27
[7]     L←(8×Q×S)-(2×Q*3)+9×R*2
[8]     Δ←-(4×P*3)+27×CQ*2
[9]     T←(Δ>0)∧(Q≥0)∨L≤0
[10]    →(LA,LB)[T+1]
[11]  LA:Δ←+/((Δ≤0),(Q<0)∨L>0)×|Δ,(|Q×Q<0)⌊L×L>0
[12]    →0
[13]  LB:Δ←0
      ∇
```

```
      ∇ C←A CROSS B
[1]     C←1⊖(A×1⌽B)-B×1⊖A
      ∇
```

```
      ∇ PRINT X
[1]     ''
[2]     'ITERATION NUMBER ';M
[3]     'CURRENT DESIGN VECTOR ';9↑X
[4]     'CURRENT STEP VECTOR ';¯9↑X
[5]     'OBJECTIVE FUNCTION VALUE ';FUN 9↑X
[6]     'TR=';1↑(TR1,TR2)[⍋TR1,TR2]
[7]     'CONSTRAINT CONDITION VALUES'
[8]     'CRT=';CRT1+CRT2
[9]     'BAT=';BAT1+BAT2
[10]    'LLRT=';LLRT1+LLRT2
[11]    'FPLT=';FPLT
[12]    'TOTAL NUMBER OF FUNCTIONAL EVALUATIONS=';FE
[13]    ''
      ∇
```

```
      ∇ RJ←EULER ANG;C;S
[1]     ANG←ANG×○÷180
[2]     C←2○ANG
[3]     S←1○ANG
[4]     RJ←((C[1]×C[3])-S[1]×S[3]×C[2])
[5]     RJ←RJ,(¯1×(C[1]×S[3])+S[1]×C[3]×C[2]),S[1]×S[2]
[6]     RJ←RJ,((S[1]×C[3])+C[1]×C[2]×S[3])
[7]     RJ←RJ,((¯1×S[1]×S[3])+C[1]×C[2]×C[3]),(¯1×C[1]×S[2])

[8]     RJ←(3 3)⍴RJ,(S[2]×S[3]),(S[2]×C[3]),C[2]
      ∇
```

Programs (continued)

```
     ∇ Z←QUART X;F;A;D2;D;V;⎕IO
[1]    ⎕IO←1
[2]    X←X×1≠1+X←1↓X÷X[1]
[3]    A←-.25×X[1]
[4]    →(((1+X[4]),(0∧.=×|X[1 3]),1+X[1])=1+F←0)/STOP,OUT,G
       O
[5]    X←0,(A⊥6 3 1×1,X[1 2]),(A⊥4 3 2 1×1,X[1 2 3]),A⊥1,X
[6]    →((1+X[4 3])=F←1)/STOP,OUT
[7]   GO:Z←CUBIC 1,(2×X[2]),(-/X[2 4]×X[2],4),-X[3]*2
[8]    D←(D2←1↑(Z[;1]>0)/(1=1+Z[;2])/Z[;1])*.5
[9]    Z←(QUAD 1,D,.5×X[2]+D2-X[3]÷D),[1]QUAD 1,(-D),.5×X[2
       ]+D2+X[3]÷D
[10]   →OUT+2
[11]  STOP:Z←0,[1]CUBIC 1,X[1 2 3]
[12]   →(F,1)/OUT+2 3
[13]  OUT:Z←2 CROOT QUAD 1,X[2 4]
[14]   →(F,1)/OUT+2 3
[15]   Z←Z+4 2ρA,0
     ∇
```

```
     ∇ Z←CUBIC X;A;B;C;D;Q;R;T;0;⎕IO
[1]    →9×⍳1=(⎕IO←1)+0⊥X
[2]    X←(A←X[1]),(B←X[2]÷3),(C←X[3]÷3),D←X[4]
[3]    →6×⍳(Q=0)∨0≤T←((Q←(A×C)-B×B)*3)+(R←(-B*3)+.5×(×/3,A,
       B,C)-×/A,A,D)*2
[4]    Z←(3 1ρ((2××¯.5+R≥0)×(Q*.5)×2○((÷3)×¯2○(|R)÷(Q←-Q)*1
       .5)+0 1 2×○2÷3)),0
[5]    →7,,Z←(Z-3 2ρB,0)÷A
[6]    Z←3 2ρ(÷A)×(-B-+/C),0,Q,R,(Q←(-.5×+/C)-B),-R←.5×(3*.
       5)×-/C←(R+T,-T←T*.5)*÷3
[7]    Z←Z×1≠1+Z
[8]    →0
[9]    Z←0,[1]QUAD X[1 2 3]
     ∇
```

```
     ∇ Z←QUAD X;⎕IO
[1]    ⎕IO←1
[2]    →4×⍳0≥Z←-(⎕CT<|Z)×Z←(4×X[3])-(X←X÷X[1])[2]*2
[3]    →0,,Z←(2 1ρ.5×(-X[2]-Z*.5),-X[2]+Z*.5),0
[4]    Z←2 2ρ.5×X,(-Z),(X←-X[2]),Z←-(-Z)*.5
     ∇
```

Programs (continued)

```
     ∇ Z←FL CROOT F;I
[1]    Z←0 2⍴I←¯1+⍳1
[2]    Z←Z,[1]FL CR,F[I←I+1;]
[3]    →2×I<1↑⍴F
     ∇


     ∇ Z←N CR X;T
[1]    Z←(((+/X*2)*.5)*÷N)×⍉(2,N)⍴(2○T),1○T←(÷N)×((○2)|(1↓X
       )ATAN 1↑X)+○2×¯1+⍳N
     ∇


     ∇ Z←Y ATAN X;X1;S;X0;IZ;⎕IO
[1]    ⎕IO←1
[2]    S←⍴X
[3]    X1←X+(X←,X)=0
[4]    Z←¯3○(Y←,Y)÷X1
[5]    Z[X0/IZ←⍳⍴Z]←.5×○×(X0←X=0)/Y
[6]    Z[X1]←Z[X1←X0/IZ]+○×(X0←X<0)/Y
[7]    Z[(X0∧Y=0)/IZ]←○1
[8]    Z←S⍴Z
     ∇
```

RSSR-SR Mechanism Optimization Sample Run

```
        0
 0  0  0
 1  1  1
 1  2  3

        R2
 .86805 ¯.49444 _.04494
 .48852  .83449 ¯.25489
 .08852  .24321  .96593

        R3
 .80393 ¯.59045 _.07111
 .57688  .74517 ¯.33455
 .14454  .30998  .93969


       HOOKE
INPUT STARTING POINT
⎕:
      8 ¯3 3 7 5 ¯2 8 ¯6 3
INPUT INITIAL STEP SIZE VECTOR
⎕:
      1 1 1 1 1 1 1 1 1
INPUT MINIMUM STEP SIZE
⎕:
      .05
INPUT PENALTY PARAMETER VALUE
⎕:
      100
INPUT MAXIMUM NUMBER OF ITERATIONS
⎕:
      25
INPUT THE NUMBER OF ITERATIONS BETWEEN PRINTINGS
⎕:
      2

ITERATION NUMBER 1
CURRENT DESIGN VECTOR 8 ¯3 3 7 5 ¯2 8 ¯6 3
CURRENT STEP VECTOR 1 1 1 1 1 1 1 1 1
OBJECTIVE FUNCTION VALUE 71.056
TR=.54516
CONSTRAINT CONDITION VALUES
CRT=1.3868
BAT=0
LLRT=26.087
FPLT=31.646
TOTAL NUMBER OF FUNCTIONAL EVALUATIONS=1
```

Sample Run (continued)

```
ITERATION NUMBER 2
CURRENT DESIGN VECTOR 6 ¯3 1 5 5 ¯2 6 ¯6 1
CURRENT STEP VECTOR ¯1.2 ¯.5 ¯1.2 ¯1.2 ¯.5 ¯.5 ¯1.2 ¯.5
      ¯1.2
OBJECTIVE FUNCTION VALUE ¯.03536
TR=.03536
CONSTRAINT CONDITION VALUES
CRT=0
BAT=0
LLRT=0
FPLT=0
TOTAL NUMBER OF FUNCTIONAL EVALUATIONS=39


ITERATION NUMBER 4
CURRENT DESIGN VECTOR 8.4 ¯.8 ¯1.4 6.2 2.8 .2 7.2 ¯6.5
      ¯.2
CURRENT STEP VECTOR ¯.72 .72 .72 .72 ¯.72 .72 .72 ¯.3
      ¯.72
OBJECTIVE FUNCTION VALUE ¯.60477
TR=.60477
CONSTRAINT CONDITION VALUES
CRT=0
BAT=0
LLRT=0
FPLT=0
TOTAL NUMBER OF FUNCTIONAL EVALUATIONS=102


ITERATION NUMBER 6
CURRENT DESIGN VECTOR 8.4 ¯1.52 ¯1.4 6.2 2.08 .2 7.2
      ¯6.5 ¯.2
CURRENT STEP VECTOR ¯.18 .432 .18 .18 .432 .18 .18
      ¯.075 ¯.18
OBJECTIVE FUNCTION VALUE ¯.66116
TR=.66116
CONSTRAINT CONDITION VALUES
CRT=0
BAT=0
LLRT=0
FPLT=0
TOTAL NUMBER OF FUNCTIONAL EVALUATIONS=175
```

Sample Run (continued)

```
ITERATION NUMBER 8
CURRENT DESIGN VECTOR 8.4 ¯1.52 ¯1.4 6.2 2.08 .2 7.2
      ¯6.5375 ¯.2
CURRENT STEP VECTOR ¯.045 .108 .045 .045 .108 .045 .045
      ¯.045 ¯.045
OBJECTIVE FUNCTION VALUE ¯.66207
TR=.66207
CONSTRAINT CONDITION VALUES
CRT=0
BAT=0
LLRT=0
FPLT=0
TOTAL NUMBER OF FUNCTIONAL EVALUATIONS=250

*****SOLUTION HAS CONVERGED*****

ITERATION NUMBER 9
CURRENT DESIGN VECTOR 8.355 ¯1.52 ¯1.4 6.2 2.08 .2 7.2
      ¯6.5375 ¯.2
CURRENT STEP VECTOR .027 .027 .01125 .01125 .027 .01125
      .01125 ¯.01125 ¯.01125
OBJECTIVE FUNCTION VALUE ¯.66329
TR=.66329
CONSTRAINT CONDITION VALUES
CRT=0
BAT=0
LLRT=0
FPLT=0
TOTAL NUMBER OF FUNCTIONAL EVALUATIONS=323
```

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BIOGRAPHICAL SKETCH

Charles F. Reinholtz was born to Theodore and Mary Reinholtz on 21 March 1954 in New Rochelle, New York. In 1958, his family moved to Brevard County, Florida, where Charles attended primary and secondary school. He received a high school diploma from Merritt Island High School in 1972, and an Associate of Arts degree from Brevard Community College in 1974. In 1976, he received a Bachelor of Science degree with honors in mechanical engineering from the University of Florida.

Charles enrolled in graduate school at the University of Florida in 1977. After a brief industrial career with Burroughs Corporation, he received the degree of Master of Engineering in mechanical engineering in 1980. He is currently pursuing the Doctor of Philosophy degree in mechanical engineering at the University of Florida.

Charles was married to the former Ms. Jeri Sue Thompson in December, 1978. They are blessed with one child, Nicholas Stephen, born 21 June 1983.

Charles is a member of the American Society of Mechanical Engineering, Tau Beta Pi and Pi Tau Sigma honor societies and Epsilon Lamda Chi Engineering Leadership Circle.

I certify that I have read this study and that in my opinion it conforms to acceptable standards of scholarly presentation and is fully adequate, in scope and quality, as a dissertation for the degree of Doctor of Philosophy.

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Professor of Mechanical Engineering

I certify that I have read this study and that in my opinion it conforms to acceptable standards of scholarly presentation and is fully adequate, in scope and quality, as a dissertation for the degree of Doctor of Philosophy.

Joseph Duffy, Cochairman
Professor of Mechanical Engineering

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